Confidence Interval for the Population Variance

Information:

Based on a random sample (suppose it is drawn from a normal distribution or at least an approximately normal distribution), we know

- \( n \), sample size
- \( S^2 \), sample variance

The 100 \times (1 - \alpha)\% confidence interval:

\[
\frac{(n - 1)S^2}{\chi_{\alpha/2,n-1}^2} < \sigma^2 < \frac{(n - 1)S^2}{\chi_{1-\alpha/2,n-1}^2}
\]

Interpretation:

If we draw 100 such random samples, and construct the intervals following the above way, there will be 100 \times (1 - \alpha) intervals that contain the true variance.
Example:

- \( n = 25 \)
- \( S^2 = 17.7^2 \)
- 95% confidence interval (\( \alpha = 0.05 \)):

\[
\frac{(25 - 1) \times 17.7^2}{39.3641} < \sigma^2 < \frac{(25 - 1) \times 17.7^2}{12.4011}
\]

\( \Rightarrow \)

\[13.821 < \sigma < 24.623\]