Suggested Exercises for Chapter 6

1 Suppose a discrete random variable $X$ takes values of 0, 1, 4 with probabilities $1/3, 1/3, 1/3$ respectively. a). Find the mean $\mu$ and the variance $\sigma^2$; b). Find the sampling distribution of $\bar{X}$ for a random sample of $n = 2$; c). Find the mean and variance of $\bar{X}$.

2 Suppose a random sample of $n = 25$ measurement is selected from a population with mean $\mu$ and standard deviation $\sigma$. For each of the following values of $\mu$ and $\sigma$, give the value of $\mu_{\bar{X}}$ and $\sigma_{\bar{X}}$. a) $\mu = 10$ and $\sigma = 3$; b) $\mu = 100$ and $\sigma = 25$; c) $\mu = 20$ and $\sigma = 40$;

3 Suppose a discrete random variable $X$ takes values of 1, 2, 3, 8 with probabilities 0.1, 0.4, 0.4, 0.1 respectively. a). Find the mean $\mu$ and the variance $\sigma^2$; b). Find the sampling distribution of $\bar{X}$ for a random sample of $n = 2$; c). Use the result of part b) to calculate $\mu_{\bar{X}}$ and $\sigma_{\bar{X}}$. Confirm that $\mu_{\bar{X}} = \mu$ and $\sigma_{\bar{X}} = \sigma/\sqrt{2}$.

4 A random sample of $n = 100$ observations is selected from a population with $\mu = 30$ and $\sigma = 16$. a) Find $\mu_{\bar{X}}$ and $\sigma_{\bar{X}}$; b) describe the shape of the sampling distribution of $\bar{X}$; c) Find $P(\bar{X} \geq 28)$; d) find $P(22.1 \leq \bar{X} \leq 26.8)$; e) find $P(\bar{X} \geq 28.2)$;

5 A random sample of $n = 2$ observations is selected from a normal population with $\mu = 30$ and $\sigma = 16$. What kind of distribution does $\bar{X}$ have? Is it normal or does it depend on the sample size?