Suggested Exercises for Chapter 5

1 Suppose $X$ is a random variable with a uniform probability distribution with lower bound 2 and upper bound 4. Find $f(x)$, the mean, the standard deviation, $P(\mu - \sigma \leq X \leq \mu + \sigma)$, and $P(X > 2.78)$.

2 Based Uniform(2, 4), find the value of $a$ for each of the following statements:
   a. $P(X \geq a) = 0.5$; b. $P(X \leq a) = 0.2$; c. $P(X \leq a) = 0$; d. $P(2.5 \leq X \leq a) = 0.5$.

3 Find the following probabilities for the standard normal random variable $Z$:
   a. $P(Z > 1.46)$; b. $P(Z < -1.56)$; c. $P(0.67 \leq Z \leq 2.41)$; d. $P(-1.96 \leq Z \leq -0.33)$; e. $P(-2.33 \leq Z \leq 1.50)$.

4 Find a value of the standard normal random variable $Z$, call it $z_0$, such that
   a. $P(Z \leq z_0) = 0.0401$; b. $P(-z_0 \leq Z \leq z_0) = 0.95$; c. $P(-z_0 \leq Z \leq z_0) = 0.8740$; d. $P(-z_0 \leq Z \leq 0) = 0.2967$; e. $P(-2 \leq Z \leq z_0) = 0.9710$.

5 The random variable $X$ has a normal distribution with $\mu = 300$ and $\sigma = 30$. a. Find the probability that $X$ assumes a value more than 2 std. dev. from its mean; b. Find the probability that $X$ assumes a value within 1 std. dev. from its mean; c. Find the value of $X$ that represents the 80th percentile of the distribution.

6 Resource Reservation Protocol (RSVP) was originally designed to establish signaling links for stationary networks, which was applied to mobile wireless technology. A simulation study revealed that the transmission delay (measured in milliseconds) of an RSVP-linked wireless device has an approximate normal distribution with mean $\mu = 48.5$ and $\sigma = 8.5$. a. What is the probability that the transmission delay is less than 57 milliseconds? b. What is the probability that the transmission delay is between 40 and 60 milliseconds.

7 Suppose $X$ has an exponential distribution with mean $\theta = 1$. Find the following probabilities:
   a. $P(X > 1)$; b. $P(X \leq 3)$; c. $P(X > 1.5)$.

8 In NASCAR races such as Daytona 500, 43 drivers start the race; however, about 10% of the cars do not finish due to the failure of critical parts. University of Portland professors conducted a study of critical-part failures from 36 NASCAR races. The researchers discovered that the time (in hours) until the first critical-part failure is exponentially distributed with a mean of 0.10 hour. a. Find the probability that the time until the first failure is 0.2 hours or more; b. Find the probability that the time until first failure is less than 0.4.

9 Suppose there are $n$ identical machines are running. For each machine, the time until failure follows an exponential distribution with mean $\theta$. Consider the time until the first failure among all these machines. What distribution does it follow?