Problem 1 (25 pts)

Estimate the integral \( I = \int_{0}^{\pi/2} \sin x \, dx \)

A) Using Trapezoidal Integration with 10 equally spaced intervals.

B) Using Simpson's 1/3 Rule with 10 equally spaced intervals.

Fill in the following table to help with the calculations. Round all calculations to 4 places after the decimal.

<table>
<thead>
<tr>
<th>i</th>
<th>( x_i )</th>
<th>( f_i )</th>
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<tbody>
<tr>
<td>0</td>
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</table>
Problem 2 (25 pts)

The following integral is to be approximated using Trapezoidal Integration.

$$I = \int_{0}^{4} \left[ -\frac{1}{12}(x-2)^{4} + 2x^{2} \right] dx$$

Let $I_n$ represent the approximation to $I$ when the interval $(0,4)$ is subdivided into $n$ equal sub-intervals. How many sub-intervals must there be so that the error $|I - I_n|$ is at most $\frac{10^{-4}}{3}$?
Problem 3 (25 pts)

A quadratic spline is to be fit thru the following points: (0,1), (1,2), (3,3), (4,2).

The spline is given by

\[
\begin{align*}
    f(x) &= b_1x + c_1 & 0 \leq x \leq 1 \\
    & a_2x^2 + b_2x + c_2 & 1 \leq x \leq 3 \\
    & a_3x^2 + b_3x + c_3 & 3 \leq x \leq 4 
\end{align*}
\]

A system of 8 equations in the 8 unknowns \( b_1, c_1, a_2, b_2, c_2, a_3, b_3, c_3 \) can be represented by \( Ax = b \). The vectors are \( x = [b_1, c_1, a_2, b_2, c_2, a_3, b_3, c_3]^T \) and \( b = [2, 2, 3, 3, 1, 2, 0, 0]^T \)

Find the coefficient matrix \( A \).
Problem 4 (25 pts)

An unknown function generated the following data points:

(0,0), (1,1.6487), (3,13.4451)

A) Find the Newton Divided Difference interpolating polynomial \( f_2(x) \).

B) An additional data point (2,5.4366) is available. Estimate the error in \( f_2(2.5) \).

C) The data points were generated from the function \( f(x) = xe^{x/2} \). Find the true error in \( f_2(2.5) \).

D) Estimate of error in \( f_2(2.5) = \) _________

E) True error in \( f_2(2.5) = \) _________
Problem 5 (25 pts)

Solve the following system of equations by using a sequence of elementary row operations on the augmented matrix until it has been reduced to its Echelon form.

\[
\begin{align*}
    w & + & x & + & y & - & z & = & 1 \\
    2w & - & x & + & 3y & - & 2z & = & -2 \\
    w & & & + & y & - & z & = & 0 \\
    2x & & & + & y & & & = & 1
\end{align*}
\]