Homework #2

Table 1. Percentiles for common distributions.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>90%</th>
<th>95%</th>
<th>97.5%</th>
<th>99%</th>
<th>99.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>N(0,1)</td>
<td>1.282</td>
<td>1.645</td>
<td>1.960</td>
<td>2.326</td>
<td>2.576</td>
</tr>
<tr>
<td>t(9)</td>
<td>1.383</td>
<td>1.833</td>
<td>2.262</td>
<td>2.821</td>
<td>3.250</td>
</tr>
<tr>
<td>t(18)</td>
<td>1.330</td>
<td>1.734</td>
<td>2.101</td>
<td>2.552</td>
<td>2.878</td>
</tr>
<tr>
<td>$\chi^2$</td>
<td>4.6052</td>
<td>5.9915</td>
<td>7.3778</td>
<td>9.2103</td>
<td>10.5966</td>
</tr>
</tbody>
</table>

1. (30 points) A random sample of $n$ measurements was selected from a population with unknown mean $\mu$ and standard deviation $\sigma$. Calculate 95% confidence intervals for $\mu$ for each of the following situations.

   a $n = 75, \bar{x} = 28, s^2 = 12.$

   b $n = 200, \bar{x} = 102, s^2 = 22.$

   c Is the assumption that the underlying population of measurements is normally distributed necessary to ensure the validity of the confidence intervals in parts a and b? Why?

2. (10 points) A random sample of 90 observations produced a sample mean 25.9 and sample standard deviation 2.7. Construct a 99% confidence interval for the population mean.

3. (10 points) Suppose you decide to estimate the mean resale value of a particular car model. Suppose the resale values follow a normal distribution. You manage to obtain data on 10 recently resold cars of that model. These 10 cars were resold at an average price of $\bar{x} = $13,700 with a sample standard deviation of $s = $800. Create a 95% confidence interval for the true mean resale value of that model.
4. (20 points) let \( t_0 \) be a specific value of \( t \). Use Table VI (pp. 796) or any statistical software to find \( t_0 \) values such that the following statements are true:

- \( P(t \geq t_0) = 0.025 \), where \( df = 10 \).
- \( P(t \geq t_0) = 0.01 \), where \( df = 17 \).
- \( P(t \leq t_0) = 0.005 \), where \( df = 6 \).
- \( P(t \leq t_0) = 0.05 \), where \( df = 13 \).

5. (10 points) The U.S. Commission on Crime wishes to estimate the fraction of crimes related to firearms in an area with one of the highest crime rates in the country. The commission randomly selects 600 files of recently committed crimes in the area and finds 380 in which a firearm was reportedly used. Find a 90% confidence interval for \( p \), the true fraction of crimes in the area in which some type of firearm was reportedly used.

6. (20 points) In each case, find the approximate sample size \( n \) required to construct a 95% confidence interval for a proportion \( p \) that has sampling error \( SE = 0.06 \).

- \( a \) Assume that \( p \) is near 0.3.

- \( b \) No prior knowledge of \( p \).