

Midterm Exam I

Instructions: This is a closed-book exam. However, you are allowed to bring two two-sided cheat sheets. Arrange your time wisely and Show all your work. Details and explanations should be given in order to receive partial/full credit. Please do well. Remember to put down your name on each page.

A random sample consisting of 5 people who drove automobiles was selected to see if alcohol affected reaction time. Each driver's reaction time was measured in a laboratory before and after drinking a specified amount of a beverage containing alcohol. The reaction times in seconds were as follows. The reaction times after drinking is of particular interest.

Subject	Before (X)	After (Y)
1	.68	.73
2	.64	.66
3	.68	.76
4	.82	.80
5	.58	.68

Note that $\sum x_i = 3.4$, $\sum y_i = 3.63$, $\sum x_i y_i = 2.4860$, $\sum x_i^2 = 2.3432$, $\sum y_i^2 = 2.6485$.

1. Assume that the reaction times after drinking (Y) are *normally* distributed. Give a 95% confidence interval for a person's average reaction time after drinking.

2. Plot the data as a scatter diagram. Does a linear model appear reasonable?

3. Suppose that simple linear regression is used to assess the relationship between the before and after reaction times of X and Y . Namely,

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i,$$

where $\epsilon_i \sim N(0, \sigma^2)$. Compute the least squares (LS) estimates of the regression parameters, $\hat{\beta}_0$ and $\hat{\beta}_1$. Then add the fitted straight line to the scatterplot in part (1).

4. Complete the following ANOVA table. Hint: for blank (5), use $SST = \sum_{i=1}^n (y_i - \bar{y})^2 = \sum_{i=1}^n y_i^2 - n \cdot \bar{y}^2$.

Source	Df	SS	MS	F Value	P-value
Model	(1)	0.009928	(6)	(8)	0.05522
Error	(2)	(4)	(7)		
Total	(3)	(5)			

5. Based on the above ANOVA table. Answer the following questions.

(a) Give an estimate for σ^2 , $\hat{\sigma}^2$.

(b) Compute estimates for the standard errors of $\hat{\beta}_0$ and $\hat{\beta}_1$.

(c) Is a person's normal reaction time (X) useful in predicting his reaction time after drinking (Y)?
($\alpha = 0.10$)

(d) Compute the coefficient of determination, R , and interpret it.

6. (**Extra Credit:** 10 points) Suppose it is known to you that $(n - 2) \cdot \frac{\hat{\sigma}^2}{\sigma^2}$ follows a χ^2 distribution with $n - 2$ degrees of freedom. That is

$$(n - 2) \cdot \frac{\hat{\sigma}^2}{\sigma^2} \sim \chi^2_{(n-2)}.$$

Construct a 95% confidence interval for σ^2 .

7. Given a person with normal reaction time $X_0 = .65$ second, construct a 95% prediction interval for his reaction time after drinking?

Note:

t	$t^{(4)}(.95) = 2.132$	$t^{(4)}(.975) = 2.776$	$t^{(3)}(.95) = 2.353$	$t^{(3)}(.975) = 3.182$
χ^2	$\chi^{(4)}(.95) = 9.48773$	$\chi^{(4)}(.975) = 11.1433$	$\chi^{(3)}(.95) = 7.81473$	$\chi^{(3)}(.975) = 9.34840$
F	$F^{(1,3)}(.90) = 5.54$	$F^{(1,3)}(.95) = 10.13$	$F^{(1,4)}(.90) = 4.45$	$F^{(1,4)}(.95) = 7.71$