Solutions for Sample Midterm 1

1. **Solution**: First compute

$$\bar{y} = \frac{\sum y_i}{n} = \frac{3.63}{5} = 0.726$$

and

$$\hat{\sigma}_y^2 = \frac{\sum y_i^2 - n\bar{y}^2}{n-1} = \frac{2.649 - 5 \times .726}{5-1} = 0.00328$$

Then the 95% CI for a person's average reaction time after drinking is

$$\bar{y} \pm t_{0.975}^{(n-1)} \frac{\hat{\sigma}_y}{\sqrt{n}}$$

, i.e.,

$$0.726 \pm 2.776 \times \sqrt{\frac{.00328}{5}} \quad \text{or} \quad (0.655,\ 0.797).$$

2. **Solution**: Omitted.

3. **Solution**:

$$\hat{\beta}_1 = \frac{\sum x_i y_i - \sum x_i \sum y_i/n}{\sum x_i^2 - (\sum x_i)^2/n} = \frac{2.486 - 3.4 \times 3.63/5}{2.343 - 3.4^2/5} = 0.564$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = \frac{3.63}{5} - 0.564 \times \frac{3.4}{5} = 0.342.$$

4. **Solution**: First compute

$$SST = \sum y_i^2 - \frac{(\sum y_i)^2}{n} = 2.6485 - \frac{3.63^2}{5} = 0.01312$$

Terms	Df	SS	MS	F	P-value
Model	1	0.009928	0.00992821	9.33162	0.05522
Error	3	0.003192	0.00106393		
Total	4	0.013120			

5. **Solution**:

(a) $\hat{\sigma}^2 = MSE = 0.00106393$;

(b)

$$s.e.(\hat{\beta}_1) = \sqrt{\frac{MSE}{\sum x_i^2 - n \cdot \bar{x}^2}} = \sqrt{\frac{0.00106393}{2.343 - 5 \times 0.68^2}} = 0.185$$

$$s.e.(\hat{\beta}_0) = \sqrt{MSE \cdot \left[\frac{1}{n} + \frac{\bar{x}^2}{\sum x_i^2 - n \cdot \bar{x}^2}\right]} = \sqrt{0.00106393 \cdot \left[\frac{1}{5} + \frac{0.68^2}{2.343 - 5 \times 0.68^2}\right]} = 0.126.$$

(c) Yes. Because the p-value is 0.05522, which is less than $\alpha = 0.10$.

(d)

$$R^2 = \frac{SSR}{SST} = \frac{0.009928}{0.013120} = 75.7\%.$$

Therefore, about 75.7% of the total variation in a person's reactime time after drinking (y) can be explained by its regression on his reaction time without drinking (x).

6. **Solution**: Since

$$(n-2) \cdot \frac{\hat{\sigma}^2}{\sigma^2} \sim \chi^2(n-2),$$

it follows that, letting $\chi^2_{0.025}(n-2)$ and $\chi^2_{.975}(n-2)$ be the 2.5-th and 97.5-th percentiles from $\chi^2(n-2)$, respectively,

$$\Pr\left(\chi^2_{0.025}(n-2) \le \frac{\hat{\sigma}^2}{\sigma^2} \le \chi^2_{.975}(n-2)\right) = 95\%$$

$$\Pr\left(\frac{(n-2) \cdot \hat{\sigma}^2}{\chi^2_{0.025}(n-2)} \ge \sigma^2 \ge \frac{(n-2) \cdot \hat{\sigma}^2}{\chi^2_{0.975}(n-2)}\right) = 95\%$$

Namely, the 95% CI for σ^2 is

$$\left(\frac{(n-2) \cdot \hat{\sigma}^2}{\chi^2_{0.025}(n-2)}, \quad \frac{(n-2) \cdot \hat{\sigma}^2}{\chi^2_{0.975}(n-2)}\right).$$

7. **Solution**: Given $x_0 = 0.65$, the 95% prediction interval is

$$(\hat{\beta}_0 + \hat{\beta}_1 \cdot x_0) \pm t_{0.975}^{(n-2)} \cdot \sqrt{MSE \cdot \left[1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{\sum x_i^2 - n \cdot \bar{x}^2}\right]}$$

$$(0.342 + 0.564 \times 0.65) \pm 3.182 \times \sqrt{0.00106393 \times \left[1 + \frac{1}{5} + \frac{(0.65 - 0.68)^2}{2.343 - 5 \times 0.68^2}\right]}$$

i.e.

$$(0.5935, \quad 0.8237).$$