

Homework 2

Problems from the textbook:

Chp3 Problems: 2, 7, 16, 25, 39, 45, 49, 71

Extra Problems:

1. Suppose that we fit the model $y_i = \mu + \varepsilon_i$, where $\varepsilon_i \rightarrow_{i.i.d.} N(0, \sigma^2)$, to a random sample Y_1, \dots, Y_n with the LS criterion. Show that the LS estimate of μ is the sample average, namely, $\hat{\mu} = \bar{y}$.
2. Prove the following identities:

$$\begin{aligned} \sum_{i=1}^n (x_i - \bar{x})^2 &= \sum_{i=1}^n x_i^2 - n \cdot \bar{x}^2 = \sum_{i=1}^n x_i^2 - \frac{\left(\sum_{i=1}^n x_i \right)^2}{n} = (n-1) \cdot s_x^2 \\ \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y}) &= \sum_{i=1}^n x_i y_i - n \cdot \bar{x} \cdot \bar{y}, \text{ and} \\ \sum_{i=1}^n (x_i - \bar{x}) \cdot (y_i - \bar{y}) &= \sum_{i=1}^n (x_i - \bar{x}) \cdot y_i. \end{aligned}$$

3. Consider the simple linear model, $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$ with $\varepsilon_i \sim_{i.i.d.} N(0, \sigma^2)$. Let $\hat{\beta}_0$ and $\hat{\beta}_1$ denote the LS estimates of the two regression parameters. Now we define the model *residuals* as follows: $e_i = y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i)$, $i = 1, \dots, n$. Show that

$$\sum_{i=1}^n e_i = 0 \text{ and } \sum_{i=1}^n x_i \cdot e_i = 0.$$