

Potencies (dosages of death) of
four cardiac substances

	1	2	3	4
	29	17	17	18
	28	25	16	20
	23	24	21	25
	26	19	22	24
	26	28	23	16
	19	21	18	20
	25	20	20	20
	29	25	17	17
	26	19	25	19
	28	24	21	17
size (n_i)	10	10	10	10
mean (\bar{y}_i)	25.9	22.2	20.0	19.6
variance (s_i^2)	9.4333	12.1778	8.6667	8.7111

Note: the grand mean is $\bar{y}_{..} = 21.92$ and variance $s_y^2 = 3.9249$.

An Illustration of Multiple Comparisons
in One-Way ANOVA

Regression Approach with Dummy Variables

Dummy variables ($Z_1 - Z_3$) are defined to fit the multiple regression model

$$y_i = \beta_0 + \beta_1 z_{i1} + \beta_2 z_{i2} + \beta_3 z_{i3} + \varepsilon_i,$$

where $\varepsilon_i \stackrel{iid}{\sim} N(0, \sigma^2)$ for $i = 1, 2, \dots, N$.

$$Z_{i1} = \begin{cases} 1 & \text{if substance 2 was infused into the } i\text{-th pig.} \\ 0 & \text{if any other substance was infused into the } i\text{-th pig.} \end{cases}$$

$$Z_{i2} = \begin{cases} 1 & \text{if substance 3 was infused into the } i\text{-th pig.} \\ 0 & \text{if any other substance was infused into the } i\text{-th pig.} \end{cases}$$

$$Z_{i3} = \begin{cases} 1 & \text{if substance 4 was infused into the } i\text{-th pig.} \\ 0 & \text{if any other substance was infused into the } i\text{-th pig.} \end{cases}$$

The Potency Data - (Page 444)

The following data were collected from an experiment designed to compare the relative potencies of four cardiac substances. In the experiment, a suitable dilution of one of the substances was slowly infused into an anesthetized guinea pig, and the dosage at which the pig died was recorded. Ten guinea pigs were used for each substance.

2. The half-length of the confidence intervals, i.e.,

$t_{1-\alpha/2}^{(n-c)} \cdot \sqrt{\text{MSE} \cdot \left(\frac{1}{n_i} + \frac{1}{n_{i'}}\right)}$, remain the same because the design is *balanced*.

Individual 95% <i>t</i> CI's			
Term	$\bar{y}_i - \bar{y}_{i'}$	L	U
2 vs 1	-3.7	-6.532	-0.868
3 vs 1	-5.9	-8.732	-3.068
4 vs 1	-6.3	-9.132	-3.468
3 vs 2	-2.2	-5.032	0.632
4 vs 2	-2.6	-5.432	0.232
4 vs 3	-0.4	-3.232	2.432

Bonferroni Simultaneous Intervals

Bonferroni simultaneous confidence intervals depends on the number of intervals to be constructed. Suppose that we want to construct Bonferroni 95% simultaneous intervals for all paired comparisons. There are a total number of $g = \binom{4}{2} = 6$ inferences. First compute

$$\alpha' = \alpha/g = 0.05/6 = 0.0083.$$

Then construct

$$(\bar{y}_i - \bar{y}_{i'}) \pm t_{1-\alpha'/2}^{(n-c)} \cdot \sqrt{\text{MSE} \cdot \left(\frac{1}{n_i} + \frac{1}{n_{i'}}\right)},$$

where $t_{1-\alpha'/2}^{(n-c)} = t_{0.99585}^{(36)} = 2.79$. So the half-length of each

Fit Performance

Parameter Estimates

Term	Estimate	S.E.	<i>t</i> value	2 Pr(> <i>t</i>)
Intercept (β_0)	25.9000	0.9873	26.234	0.00000
factor 2 (β_1)	-3.7000	1.3962	-2.650	0.01188
factor 3 (β_2)	-5.9000	1.3962	-4.226	0.00016
factor 4 (β_3)	-6.3000	1.3962	-4.512	0.00007

Note what happens to the standard errors of $\hat{\beta}_1$, $\hat{\beta}_2$, and $\hat{\beta}_3$.

Analysis of Variance Table

Source	df	SS	MS	<i>F</i>	Pr(> <i>F</i>)
Model	3	249.875	83.29	8.545	0.00020
Error	36	350.900	9.747		
Total	39	600.775			

Individual *t* Intervals

The 95% CI for individual paired comparisons can be constructed as

$$(\bar{y}_i - \bar{y}_{i'}) \pm t_{1-\alpha/2}^{(n-c)} \cdot \sqrt{\text{MSE} \cdot \left(\frac{1}{n_i} + \frac{1}{n_{i'}}\right)}$$

$$(\bar{y}_i - \bar{y}_{i'}) \pm 2.028 \times \sqrt{9.747 \times \left(\frac{1}{10} + \frac{1}{10}\right)}$$

$$(\bar{y}_i - \bar{y}_{i'}) \pm 2.832.$$

Note that

1.

$$\text{MSE} = \frac{(n_1 - 1) \cdot s_1^2 + \dots + (n_c - 1) \cdot s_c^2}{n - c},$$

which can be computed solely based on the information given in slide 3.

Scheffé's Simultaneous Intervals

Scheffé's method is generally recommended when comparisons other than pairwise comparisons are of interest or when the design is unbalanced:

$$(\bar{y}_i - \bar{y}_{i'}) \pm \sqrt{(c-1) \cdot F_{1-\alpha}^{(c-1, n-c)}} \cdot \sqrt{\text{MSE} \cdot \left(\frac{1}{n_i} + \frac{1}{n_{i'}} \right)},$$

where $F_{1-\alpha}^{(c-1, n-c)} = F_{0.95}^{(4-1, 40-4)} = 2.886$. So the half-length of each confidence interval is $\sqrt{(4-1) \times 2.886 \times 1.396} = 4.108$.

Tukey's 95% CI's

Term	$\bar{y}_i - \bar{y}_{i'}$	L	U
2 vs 1	-3.7	-7.808	0.408
3 vs 1	-5.9	-10.008	-1.792
4 vs 1	-6.3	-10.408	-2.192
3 vs 2	-2.2	-6.308	1.908
4 vs 2	-2.6	-6.708	1.508
4 vs 3	-0.4	-4.508	3.708

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Question: What if we want to construct 95% Scheffé's confidence intervals for $\mu_3 - \mu_1$, $\mu_4 - \mu_1$, and $(\mu_1 + \mu_2)/2 - (\mu_3 + \mu_4)/2$ only?

$$\left(\frac{\bar{y}_1 + \bar{y}_1}{2} - \frac{\bar{y}_3 + \bar{y}_1}{4} \right) \pm \sqrt{(c-1) \cdot F_{1-\alpha}^{(c-1, n-c)}} \cdot \text{s.e.},$$

where

$$\begin{aligned} \text{s.e.} &= \sqrt{\text{MSE} \cdot \left(\frac{(1/2)^2}{10} + \frac{(1/2)^2}{10} + \frac{(-1/2)^2}{10} + \frac{(-1/2)^2}{10} \right)} \\ &= 0.9873. \end{aligned}$$

Hence the confidence interval is $4.25 \pm 2.905 = (1.345, 7.155)$.

Note that both Tukey's method and Scheffé's method does not depend on the number of inferences to be made.

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confidence interval is

$$2.79 \times 1.396 = 3.895.$$

Bonferroni 95% CI's

Term	$\bar{y}_i - \bar{y}_{i'}$	L	U
2 vs 1	-3.7	-7.595	0.195
3 vs 1	-5.9	-9.795	-2.005
4 vs 1	-6.3	-10.195	-2.405
3 vs 2	-2.2	-6.095	1.695
4 vs 2	-2.6	-6.495	1.295
4 vs 3	-0.4	-4.295	3.495

Question: What if we want to construct 95% Bonferroni confidence intervals for $\mu_3 - \mu_1$, $\mu_4 - \mu_1$, and $(\mu_1 + \mu_2)/2 - (\mu_3 + \mu_4)/2$ only?

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Tukey's Simultaneous Intervals

Tukey's method works best when pairwise comparisons are of interest and the design is balanced:

$$(\bar{y}_i - \bar{y}_{i'}) \pm \frac{q_{1-\alpha}^{(c, n-c)}}{\sqrt{2}} \cdot \sqrt{\text{MSE} \cdot \left(\frac{1}{n_i} + \frac{1}{n_{i'}} \right)},$$

where $q_{1-\alpha}^{(c, n-c)} = q_{0.95}^{(4, 40-4)} = 3.81$. So the half-length of each confidence interval is $3.81/\sqrt{2} \times 1.396 = 3.762$.

Tukey's 95% CI's

Term	$\bar{y}_i - \bar{y}_{i'}$	L	U
2 vs 1	-3.7	-7.462	0.062
3 vs 1	-5.9	-9.662	-2.138
4 vs 1	-6.3	-10.062	-2.538
3 vs 2	-2.2	-5.962	1.562
4 vs 2	-2.6	-6.362	1.162
4 vs 3	-0.4	-4.162	3.362

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