

Homework #1 (Chapter 0: Review)

1. Suppose that you selected a random sample of size 4 from a normal population with mean 8 and standard deviation 2.
 - a. Can you assume that the sample mean are normally distributed? Explain.
 - b. Find $P(\bar{x} > 10)$.
 - c. Find $P(\bar{x} < 7)$.

<Solutions to EX 6.4>:

- (a) Yes. The sampling distribution of sample mean is exactly normal if the sampled population is normally distributed no matter how small the sample size is.

$$\text{standard error} = \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{4}} = 1$$

$$\begin{aligned} \text{(b) } P(\bar{x} > 10) &= P\left(\frac{\bar{x} - \mu}{\sigma_{\bar{x}}} > \frac{10 - \mu}{\sigma_{\bar{x}}}\right) = P\left(z > \frac{10 - 8}{1}\right) \\ &= P(z > 2) = 0.0228 \end{aligned}$$

$$\begin{aligned} \text{(c) } P(\bar{x} < 7) &= P\left(\frac{\bar{x} - \mu}{\sigma_{\bar{x}}} < \frac{7 - \mu}{\sigma_{\bar{x}}}\right) = P\left(z < \frac{7 - 8}{1}\right) \\ &= P(z < -1) = 0.1587 \end{aligned}$$

2. An oil exploration firm plans to drill six holes. Due to experience, the probability of each hole yielding oil is 0.12. Since the holes are in quite different locations, the outcome of drilling one hole is statistically independent of drilling of any other holes.
 - a. Give the expectation and standard deviation of the number of holes that results in oil.
 - b. If the firm will be able to stay in business only if two or more holes produce oil, what is the probability that it can survive.

Solution:

a. $n = 6$ and $p = 0.12$

$$\mu = np = 6 * 0.12 = 0.72$$

$$\sigma = \sqrt{np(1-p)} = \sqrt{6 \times 0.12 \times (1 - 0.12)} = 0.796$$

b. $P(X \geq 2) =$

$$P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=6) =$$

$$0.129534197 + 0.023551672 + 0.00240869376 + 0.000131383296$$

$$+ 0.000002985984 = 0.156$$

Note:

1. We can not use the Binomial probabilities Table to obtain this probability because $p = 0.12$ is not in the Table.
2. We can obtain this probability much easier with the concept of complement event.

$$P(X \geq 2) = 1 - P(X \leq 1) = 1 - P(X=0) - P(X=1) = 1 - P(X=0) - P(X=1) = 0.156$$

Other Problems from the Text:

9.9

9.107 (a) and (b)