

| Property | Sale Price<br>( $y$ ) | Land<br>Value ( $x_1$ ) | Improvements<br>Value ( $x_2$ ) | Area<br>$x_3$ |
|----------|-----------------------|-------------------------|---------------------------------|---------------|
| 1        | 68900                 | 5960                    | 44967                           | 1873          |
| 2        | 48500                 | 9000                    | 27860                           | 928           |
| 3        | 55500                 | 9500                    | 31439                           | 1126          |
| 4        | 62000                 | 10000                   | 39592                           | 1265          |
| 5        | 116500                | 18000                   | 72827                           | 2214          |
| 6        | 45000                 | 8500                    | 27317                           | 912           |
| 7        | 38000                 | 8000                    | 29856                           | 899           |
| 8        | 83000                 | 23000                   | 47752                           | 1803          |
| 9        | 59000                 | 8100                    | 39117                           | 1204          |
| 10       | 47500                 | 9000                    | 29349                           | 1725          |
| 11       | 40500                 | 7300                    | 40166                           | 1080          |
| 12       | 40000                 | 8000                    | 31679                           | 1529          |
| 13       | 97000                 | 20000                   | 58510                           | 2455          |
| 14       | 45500                 | 8000                    | 23454                           | 1151          |
| 15       | 40900                 | 8000                    | 20897                           | 1173          |
| 16       | 80000                 | 10500                   | 56248                           | 1960          |
| 17       | 56000                 | 4000                    | 20859                           | 1344          |
| 18       | 37000                 | 4500                    | 22610                           | 988           |
| 19       | 50000                 | 3400                    | 35948                           | 1076          |
| 20       | 22400                 | 1500                    | 5779                            | 962           |

## Multiple Linear Regression

A Numerical Example on LS Estimates and ANOVA Table

$$\text{Model I: } y_i = \beta_0 + \beta_1 x_{1i} + \varepsilon_i$$

### Parameter Estimates

|             | Estimate | S.E.   | $t$ value | $2\Pr(>  t )$ |
|-------------|----------|--------|-----------|---------------|
| (Intercept) | 6.6640   | 0.8429 | 7.9056    | 0.0000        |
| $x_1$       | 0.4697   | 0.0938 | 5.0071    | 0.0001        |

### Analysis of Variance Table

|           | df | SS    | MS    | $F$    | $\Pr(> F)$    |
|-----------|----|-------|-------|--------|---------------|
| Model     | 1  | 1.595 | 1.595 | 25.071 | 0.00009142625 |
| Residuals | 18 | 1.145 | 0.064 |        |               |
| Total     | 19 | 2.740 |       |        |               |

## The Real Estate Data

Suppose a property appraiser wants to model the relationship between the sale price ( $Y$ ) of a residential property in a mid-size city and the following three predictors:

1.  $X_1$  – Appraised land value (in dollars)
2.  $X_2$  – Appraised improvements (in dollars)
3.  $X_3$  – Area (square feet)

A *logarithm* transformation was applied to each of these variables before model fitting.

### A General $F$ Test for Nested Models

Suppose the full model and the reduced model under  $H_0$  have  $k$  and  $m$ ,  $m < k$ , parameters, respectively.

$$F = \frac{(SSE_{\text{reduced}} - SSE_{\text{full}})/(k - m)}{MSE_{\text{full}}} = \frac{(SSR_{\text{full}} - SSR_{\text{reduced}})/(k - m)}{MSE_{\text{full}}} \stackrel{H_0}{\sim} F(k - m, n - k)$$

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### Model II: $y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \varepsilon_i$

#### Parameter Estimates

|             | Estimate | S.E.   | $t$ value | $2 \Pr(>  t )$ |
|-------------|----------|--------|-----------|----------------|
| (Intercept) | 4.5514   | 0.8827 | 5.1565    | 0.0001         |
| $x_1$       | 0.1284   | 0.1202 | 1.0680    | 0.3005         |
| $x_2$       | 0.4992   | 0.1397 | 3.5735    | 0.0023         |

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#### Analysis of Variance Table

|           | df | SS    | MS    | $F$    | $\Pr(> F)$ |
|-----------|----|-------|-------|--------|------------|
| Model     | 2  | 2.086 | 1.043 | 27.117 | 5.140e-006 |
| Residuals | 17 | 0.654 | 0.038 |        |            |
| Total     | 19 | 2.740 |       |        |            |

### Interaction

Model IV:  $y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{1i}x_{2i} + \varepsilon_i$

#### Parameter Estimates

|             | Estimate | S.E.  | $t$ value | $2 \Pr(>  t )$ |
|-------------|----------|-------|-----------|----------------|
| (Intercept) | 21.091   | 6.418 | 3.286     | 0.005          |
| $x_1$       | -1.905   | 0.790 | -2.410    | 0.028          |
| $x_2$       | -1.081   | 0.621 | -1.742    | 0.101          |
| $x_1x_2$    | 0.194    | 0.075 | 2.595     | 0.020          |

#### Analysis of Variance Table

|           | df | SS    | MS    | $F$  | $\Pr(> F)$ |
|-----------|----|-------|-------|------|------------|
| Model     | 3  | 2.280 | 0.760 | 26.4 | 1.95e-006  |
| Residuals | 16 | 0.460 | 0.029 |      |            |
| Total     | 19 | 2.740 |       |      |            |

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### Model III: $y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \varepsilon_i$

#### Parameter Estimates

|             | Estimate | S.E.   | $t$ value | $2\Pr(>  t )$ |
|-------------|----------|--------|-----------|---------------|
| (Intercept) | 2.9373   | 0.8790 | 3.3414    | 0.0041        |
| $x_1$       | 0.0792   | 0.0985 | 0.8038    | 0.4333        |
| $x_2$       | 0.3640   | 0.1209 | 3.0101    | 0.0083        |
| $x_3$       | 0.4810   | 0.1526 | 3.1528    | 0.0062        |

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#### Analysis of Variance Table

|           | df | SS    | MS    | $F$    | $\Pr(> F)$ |
|-----------|----|-------|-------|--------|------------|
| Model     | 3  | 2.337 | 0.779 | 30.899 | 6.860e-007 |
| Residuals | 16 | 0.403 | 0.025 |        |            |
| Total     | 19 | 2.740 |       |        |            |