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# Decision support under uncertainties based on robust Bayesian networks in reverse logistics management

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Eduard Shevtshenko\*

Department of Machinery,  
Tallinn University of Technology,  
Ehitajate tee 5, 19086, Tallinn, Estonia  
E-mail: eduard.shevtshenko@ttu.ee

\*Corresponding author

Yan Wang

Department of Industrial Engineering & Management Systems,  
University of Central Florida,  
4000 Central Florida Blvd.,  
Orlando, Florida 32816-2993, USA  
E-mail: wangyan@mail.ucf.edu

**Abstract:** One of the major challenges for product lifecycle management systems is the lack of integrated decision support tools to help decision-making with available information in collaborative enterprise networks. Uncertainties are inherent in such networks due to lack of perfect knowledge or conflicting information. In this paper, a robust decision support approach based on imprecise probabilities is proposed. Robust Bayesian belief networks with interval probabilities are used to estimate imprecise posterior probabilities in probabilistic inference. This generic approach is demonstrated with decision-makings in design for closed-loop supply chain. The ultimate goal of robust intelligent decision support systems is to enhance the effective use of information available in collaborative engineering environments.

**Keywords:** product lifecycle management; PLM; reverse logistics; interval analysis; imprecise probability; Bayesian network.

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**Biographical notes:** Eduard Shevtshenko is working in Tallinn University of Technology as a Researcher since 2003. He received his BS, MS and PhD from the Department of Mechanical Engineering at the Tallinn University of Technology (TUT). He teaches international MS courses at TUT and consults with international corporations. His principal research areas include ERP/DSS/MRP, knowledge management and collaborative enterprise networks.

Yan Wang is an Assistant Professor at the Department of Industrial Engineering and Management Systems, University of Central Florida. He received his BS from Tsinghua University, MS from Chinese Academy of Sciences and PhD from the University of Pittsburgh. His research interests include engineering design, modelling, simulation and visualisation.

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## 1 Introduction

Under the pressure from global competition, corporations have shown interests in the close cooperation with partners in the past few years. Small and medium-sized companies have particularly been determined to set up cooperation networks. The competition in business has changed from company versus company to business network versus business network (Zheng and Pospel, 2002).

Collaborative product development among designers, manufacturers, suppliers, vendors, users and other

stakeholders is one of the keys for manufacturers to improve product quality, reduce cost and shorten time-to-market in global competition. Collaborative design is the new design process where multidisciplinary stakeholders participate in design decision-making and share product information across enterprise boundaries in an internet-enabled distributed environment. New technologies for collaborative design were developed recently, such as agent system (Shen et al., 2001), collaborative environment (Sriram, 2002), information management (Huang and Mak, 2003) and intelligent system (Zha, 2007).

Product lifecycle management (PLM) systems have been widely accepted as the major enterprise-level platform for information sharing and integration in collaborative design and manufacturing. It consists of a collection of software tools including product data management (PDM), enterprise resource planning (ERP), collaboration process management (CPM), customer relationship management (CRM), supplier relationship management (SRM), document knowledge management (DKM), environment health and safety management (EHSM) and others. Yet, one of the major challenges for PLM systems is the lack of integrated decision support tools to help decision-making with available information within the systems.

This paper addresses the need of decision support in the collaborative networks of production enterprises. An intelligent decision support system (IDSS) should integrate with different ERP systems in such networks of collaborative enterprises. An IDSS is a strategic and tactical tool capable of supporting a variety of users in making informed decisions. Information from this system will be used to support both the external and internal objectives of a corporation. The role of the IDSS is to suggest solutions given certain situations. Thus human users can assess the proposals prepared by the system and make decisions. The IDSS enables enterprise networks to be less dependent on personal experiences of employees and facilitate enterprise knowledge accumulation.

The effectiveness of an IDSS is dependent on the alignment of two conditions: the ability to collect the required data from the business functions and the conversion of the data into useful information. One challenge of decision-making in such collaborative networks is *uncertainty*. Uncertainty is due to lack of perfect knowledge or enough information. It is also known as epistemic uncertainty and reducible uncertainty. There are several sources of uncertainties in collaborative networks, including:

- Lack of data: the basic function of ERP systems is to collect and share information. When collaboration is across enterprise boundaries, not all enterprise data are sharable. Sensitive parameters, trading secrets and other intellectual properties from other companies usually are not available.
- Conflicting information: if there are multiple sources of information through different ERP systems or databases, decision-makers may face conflicts among them. It is not wise to draw simple conclusions without considering the contradictory evidence.
- Conflicting beliefs: when data are not available, decision-makers usually depend on domain experts' opinions. The judgments from those experts can be different due to the diversity of their past experiences.
- Lack of introspection: decision-makers may not be able to afford the necessary time to think deliberately about an uncertain event. Lack of introspection makes decision-making inherently risky.
- Measurement errors: the data collected by the ERP systems may contain errors due to measuring environments and human errors. The quality of collected quantities affects decision-makers' judgments.

Therefore, uncertainty should be incorporated in the IDSS for enterprise networks. Traditionally, Bayesian networks are used to accommodate uncertainties in probabilistic inference. In a dynamic business environment, decision-makers usually are required to make proper decisions related to product portfolio, platform selection, material flow and others based on the latest available information. Bayesian networks are convenient in updating prior knowledge based on the extra information. They capture relationships among random variables and provide a reasoning approach with the underlying Bayes' theorem. Bayesian networks have been widely applied in classification, data fusion, information retrieval and decision support. Nevertheless, the traditional Bayesian networks do not differentiate uncertainty from *variability*. Variability is due to the inherent randomness in a system. It is irreducible even by additional measurements and extra information. Therefore, variability is different from uncertainty. The traditional Bayesian networks consider variability and uncertainty collectively and simply represent them with probability distributions.

Uncertainty in Bayesian networks are manifested as impreciseness of probability distributions due to lack of knowledge. For instance, the probability that our market share will go up in the next six months is between 0.2 and 0.4, instead of 0.3 precisely or the probability that our new product will last longer than ten years is between 0.7 and 0.8. The impreciseness directly affects the robustness of the reasoning process. This impreciseness can be interpreted as uncertain situations. In such cases, we intend to consider a range of possible scenarios, instead of one, to ensure the robustness during decision-making.

In this paper, we propose a new decision-making approach based on robust Bayesian networks under uncertainty for IDSS, where interval-valued imprecise probabilities are used. Interval values consider a range of situations and represent uncertainties. In combination with probabilities that address variabilities, imprecise probabilities with lower and upper bounds allow us to consider a range of possible scenarios simultaneously in probabilistic inference. Incorporating uncertainties in stochastic models is particularly important when the size of available data is small or contradictory evidence does not allow us to reach consensus.

In the remainder of the paper, Section 2 gives a brief overview of Bayesian network and imprecise probability. In Section 3, we present the proposed robust Bayesian belief networks (BBNs) for decision-making under uncertainties in IDSS systems. In Section 4, we apply the new probabilistic reasoning approach to a general framework of closed-loop supply chain and illustrate it with an example of circuit board lifecycle decision-making in Section 5.

## 2 Background

### 2.1 Bayesian belief network

A BBN is a probabilistic graphical model with elements of nodes, arrows between nodes and probability assignments. We can consider a Bayesian network as a directed acyclic graph in which nodes represent random variables, where the random variable may be either discrete or continuous. In the case of discrete variables, they represent finite sets of mutually exclusive states which themselves can be categorical. Bayesian networks have a built-in computational architecture for computing the effect of evidence on the states of the variables.

BBN is able to update the probabilities of variable states while learning new evidence. It also utilises probabilistic independence relationships, both explicitly and implicitly represented in graphical models, in order to compute efficiently for large and complex problems (Taroni et al., 2006).

In BBN, the decision-maker is concerned with determining the probability that a hypothesis ( $H$ ) is true, from evidence ( $E$ ) linking the hypothesis to other observed states of the world. The approach makes use of the Bayes' rule to combine various sources of evidence. The Bayes' rule states that the posterior probability of hypothesis  $H$  given that evidence  $E$  is present or  $P(H|E)$ , is

$$P(H | E) = \frac{P(E | H)P(H)}{P(E)}$$

where  $P(H)$  is the probability of the hypothesis being true prior to obtaining the evidence  $E$  and  $P(E|H)$  is the likelihood of obtaining the evidence  $E$  given that the hypothesis  $H$  is true.

When the evidence consists of multiple sources denoted as  $E_1, E_2, \dots, E_n$ , each of which is conditionally independent, the Bayes' rule can be expanded into the expression:

$$P(H | \bigcap_j E_j) = \frac{\prod_{j=1}^n P(E_j | H)P(H)}{\prod_{j=1}^n P(E_j)}$$

The BBN architecture updates probabilities of the variable states on learning new evidence.

The BBN approach has been applied in solving manufacturing and production related problems. For instance, an interesting approach of online alert systems for production plants was proposed (Nielsen and Jensen, 2007). A methodology was developed for detecting fault and abnormal behaviours in production plants. This methodology has been successfully tested on both real world data from a power plant and simulated data from an oil production facility.

BBN was applied in root cause diagnostics of process variations (Dev and Story, 2005). It is an effective tool to

explicitly address input uncertainty and utilise data from multiple sources. After being trained with data sets, the network was able to diagnose the correct state at a 60% confidence level.

BBN was also successfully implemented for technology planning (Spath and Agostini, 1997). The aim was to design a system for adaptive planning with integrated feedbacks from real time process data and experiences. A Bayesian structure was derived from historical data stored as processing elements. It was allowed to update the network (both the structure and the probabilities) and to expand, improve or optimise the decision base.

BBN has also been used as the knowledge base of reasoning systems for supply chain diagnostics and prediction, vendor appraisal, customer assessment, evaluation of strategic or technical alliance (Kao et al., 2005). The participating enterprises in the supply chain can solve the reasoning problems based on the networks.

The different applications mentioned above are based on the traditional BBN models, where probabilities are assumed to be precisely known. When this assumption does not necessarily hold, the robustness of BBN should be aware of. Sensitivity analysis is the common approach to study the effect of uncertainty by introducing the variations of probability values.

A neighbourhood concept from sensitivity analysis, called  $\varepsilon$ -contamination model, is usually used to study the robustness (Insua and Ruggeri, 2000). It is focused on replacing a single prior distribution by a class of priors. Computing the range of the ensuring answers as the prior varying over the class is based on the model.

$$\Gamma_\varepsilon = \{P : P = (1 - \varepsilon)P_0 + \varepsilon Q, Q \in \vartheta\}$$

where  $\Gamma_\varepsilon$  is the  $\varepsilon$ -neighbourhood of probability  $P_0$  and  $\vartheta$  is called the class of contaminations that contains some arbitrary probability measurement  $Q$ . The popularity of the model arises, in part, from the ease of its specification and from the fact that it can be easily handled by the traditional precise probability theory.

In this paper, we propose a different approach to incorporate uncertainty in probabilistic inference. We integrate interval-based imprecise probabilities into Bayesian networks in order to improve the robustness of reasoning. The foundation of our approach is imprecise probability, as introduced in Section 2.2.

### 2.2 Imprecise probability

Imprecise probability is a generalisation of traditional probability to differentiate uncertainty from variability both qualitatively and quantitatively. An interval-valued probability  $[\underline{p}, \bar{p}]$  with the lower and upper bounds captures imprecision and indeterminacy. The width of the interval reflects the degree of uncertainty.

There have been several representations of imprecise probabilities. For example, behavioural imprecise probability theory (Walley, 1991a) models behavioural

uncertainties with the lower prevision  $\underline{P}(X)$  (a maximal acceptable buying price for the uncertain reward  $X$ ) and the upper prevision  $\bar{P}(X) = -\underline{P}(-X)$  (a minimal acceptable selling price for  $X$ ). Coherence principles are developed to avoid sure loss and natural extension. The Dempster-Shafer evidence theory (Dempster, 1967; Shafer, 1976) characterises uncertainties by the aid of basic probability assignments  $m(A)$  associated with the focal element  $A$ . A *belief-plausibility* pair,  $\text{Bel}(A) = \sum_{B_i \subset A} m(B_i)$  and  $\text{Pl}(A) = \sum_{B_i \cap A \neq \emptyset} m(B_i)$ , are measures of uncertainty based on the collective evidence, since  $\text{Bel}(A) \leq \text{Pl}(A)$ . The possibility theory (Zadeh, 1978; Dubois and Prade, 1998) provides an alternative to represent uncertainties with *necessity-possibility* pairs. Possibility can be regarded as a special situation of the plausibility measure when all focal elements  $B_i$ 's are nested. And the corresponding special belief measure is the necessity. A random set (Molchanov, 2005) is a multi-valued mapping from the probability space to the value space. Probability bound analysis (Ferson et al., 2004) captures uncertain information with p-boxes which are pairs of lower and upper probability distributions. F-probability (Weichselberger, 2000) incorporates intervals into probability values which also maintain the Kolmogorov properties. Fuzzy probability (Möller and Beer, 2004) considers probability distributions with fuzzy parameters. A cloud (Neumaier, 2004) is a fuzzy interval with an interval-valued membership, which is a combination of fuzzy sets, intervals and probability distributions.

All of these forms treat variability and uncertainty separately and propagates them differently so that each maintains its own character during analysis. In this paper, we take an interval-valued imprecise probability approach for Bayesian networks to improve the robustness of decision-making.

### 3 Robust BBN

One may regard an IDSS with the BBN mechanism as a consultant that supplies various models and assessments, combines all judgments and finally informs the user 'if you accept all these judgments, then you should draw these conclusions'. In this process, robustness is concerned with the sensitivity of the results of Bayesian analysis with respect to the inputs.

Our proposed robust BBN is based on interval-valued imprecise probabilities with a generalised interval form (Wang, 2008, 2009). Traditionally, a set-based interval  $[a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$  is a set of real numbers defined by its lower and upper bounds. Therefore, the interval  $[a, b]$  becomes invalid or empty when  $a > b$ . A generalised interval is no longer restricted to the ordered bound condition of  $a \leq b$ . This generalisation simplifies the Bayes' rule with imprecise probabilities and its computation.

An interval probability captures uncertainties in stochastic processes by simultaneously considering a set of probabilities. The interval probability of event  $A$  is defined as:

$$P(A) := [\underline{P}(A), \bar{P}(A)] \quad (0 \leq \underline{P}(A) \leq \bar{P}(A) \leq 1) \quad (1)$$

with its lower bound  $\underline{P}$  and upper bound  $\bar{P}$ . In the case  $\underline{P}(A) = \bar{P}(A)$ , the degenerated interval probability  $P(A)$  becomes a traditional precise probability.

The foundation of our imprecise probability is that all imprecise probabilities are subject to a *logic coherence constraint*. That is, the imprecise probabilities of event  $A$  and its complement  $A^c$  have the relationship

$$\begin{cases} \underline{P}(A) + \bar{P}(A^c) = 1 \\ \bar{P}(A) + \underline{P}(A^c) = 1 \end{cases} \quad (2)$$

The logic coherence constraint greatly simplifies the probabilistic calculus structure.

For a class of decision problems, there exists a sequence of Bayesian decision problems whose solutions converge towards the robust solution. This holds independently of whether the preference for robustness is global or restricted to local perturbations around some reference model. It is shown, that there is a sequence of Bayesian decision problems with ever increasing risk aversion with the associated optimal decisions converging to the optimal robust decision (Adam, 2004). It means that, it is possible to achieve solution which is very close to the optimal decision even when there is no precise prior information.

Robust BBN allows us to find solutions under the conditions when prior probabilities are not known exactly. This solution will also be acceptable in majority of the cases after the precise information is obtained. In other words, the robust decision needs to be made before the precise prior information will be available. The practical motivation underlying the robust Bayesian analysis is the difficulty in assessing the accuracy of prior probability distributions. Robustness with respect to prior distributions stems from the practical impossibility of eliciting a unique and precise distribution. Similar concerns apply to the other elements (likelihood and loss functions) considered in Bayesian analysis. The main goal of Bayesian robustness is to quantify and interpret the uncertainties induced by partial knowledge of one (or more) of the three elements in the analysis.

Given uncertainties involved in prior probabilities, the estimation of imprecise posterior probabilities is based on the generalised Bayes' rule (GBR) (Walley, 1996; Wang, 2008)

$$\underline{P}(A|B) = \frac{\underline{P}(B|A)\underline{P}(A)}{\underline{P}(B|A)\underline{P}(A) + \bar{P}(B|A^c)\bar{P}(A^c)} \quad (3)$$

$$\bar{P}(A|B) = \frac{\bar{P}(B|A)\bar{P}(A)}{\bar{P}(B|A)\bar{P}(A) + \underline{P}(B|A^c)\underline{P}(A^c)} \quad (4)$$

The GBR gives the lower and upper bounds of all possible posterior probabilities given the ranges of prior probabilities. It is used to update the initial estimation of  $P(A)$  after learning that event  $B$  has occurred. The upper and lower probabilities,  $\bar{P}$  and  $\underline{P}$ , are specified for all subsets of the sample space. We wish to construct posterior upper and lower probabilities  $\bar{P}(\cdot|B)$  and  $\underline{P}(\cdot|B)$ , i.e., to update beliefs after observing the new evidence.

If there are two sources of new evidence from events  $A$  and  $B$ , the assessment of event  $C$  can be based on a more general structure. In the traditional BBN,

$$P(C|A,B) = \frac{P(A|C)P(B|C,A)P(C)}{\left[ \begin{array}{l} P(A|C)P(B|C,A)P(C) + \\ P(A|C^c)P(B|C^c,A)P(C^c) \end{array} \right]} \quad (5)$$

where the precise probabilities are used. We extend the posterior probability estimation in equation (5) to consider imprecise probabilities as

$$\underline{P}(C|A,B) = \frac{\underline{P}(A|C)\underline{P}(B|C,A)\underline{P}(C)}{\left[ \begin{array}{l} \underline{P}(A|C)\underline{P}(B|C,A)\underline{P}(C) + \\ \bar{P}(A|C^c)\bar{P}(B|C^c,A)\bar{P}(C^c) \end{array} \right]} \quad (6)$$

$$\bar{P}(C|A,B) = \frac{\bar{P}(A|C)\bar{P}(B|C,A)\bar{P}(C)}{\left[ \begin{array}{l} \bar{P}(A|C)\bar{P}(B|C,A)\bar{P}(C) + \\ \underline{P}(A|C^c)\underline{P}(B|C^c,A)\underline{P}(C^c) \end{array} \right]} \quad (7)$$

In BBN, if  $A$  and  $B$  are conditionally independent, then equations (6) and (7) can be simplified as

$$\underline{P}(C|A,B) = \frac{\underline{P}(A|C)\underline{P}(B|C)\underline{P}(C)}{\left[ \begin{array}{l} \underline{P}(A|C)\underline{P}(B|C)\underline{P}(C) + \\ \bar{P}(A|C^c)\bar{P}(B|C^c)\bar{P}(C^c) \end{array} \right]} \quad (8)$$

$$\bar{P}(C|A,B) = \frac{\bar{P}(A|C)\bar{P}(B|C)\bar{P}(C)}{\left[ \begin{array}{l} \bar{P}(A|C)\bar{P}(B|C)\bar{P}(C) + \\ \underline{P}(A|C^c)\underline{P}(B|C^c)\underline{P}(C^c) \end{array} \right]} \quad (9)$$

respectively.

After new evidence has taken place, we can compare if the upper and lower probabilities of our final goal, such as the success of a new product development project, are increased or decreased. If it is decreased, we must respond with a corrective action, which is able to increase the probability of success. A collection of corrective actions may be required. We would like to see how the final result will be changed with different actions. For instance, in collaborative design, the original equipment manufacturer (OEM) makes decisions on design parameters and configurations based on the life expectancies of components from suppliers. Designers may need to choose one of the available cooling fans with different sizes and speeds based on the reliability of circuit boards from suppliers. When the suppliers provide different sets of inconsistent data, we make decisions based on the collective information, as well

as our past knowledge about the probability of successful design implementation. Nevertheless, when further information is received from suppliers, we need to respond with corrective actions and may choose a different design. It is important to find the optimal range of corrective actions, which will enable us to achieve the required upper and lower probabilities of final goal. The higher the probability of final goal is, the higher the utility of actions performed will be.

Upper and lower probabilities are used to compare actions and make decisions in the following way. In Walley (1991b), they are referred to as upper and lower provisions respectively. A decision-maker's lower prevision is the highest price at which the decision-maker is sure, he or she would bet or buy a gamble and the upper prevision is the lowest price at which the decision-maker is sure, he or she would buy the opposite of the gamble. Suppose that we need to choose an action from a finite set of possible actions  $\{a_1, a_2, \dots, a_k\}$ , where the utility  $U(a, \omega)$  of action  $a$  depends on the unknown situation  $\omega$ . We assume that utilities are specified precisely. Otherwise, the decision problem is much more complicated. Define a corrective action reward  $X_j$  by  $X_j(\omega) = U(a_j, \omega)$  for each  $j = 1, 2, \dots, k$ . To compare two possible corrective actions  $a_i$  and  $a_j$  for proper decision-making, we compute the upper and lower provisions  $\bar{P}(X_i - X_j)$  and  $\underline{P}(X_i - X_j)$  based on available information. Then, action  $a_i$  is preferred to  $a_j$  if  $\underline{P}(X_i - X_j) > 0$ . On the other hand,  $a_j$  is preferred to  $a_i$  if  $\bar{P}(X_i - X_j) < 0$ . If neither of the conditions hold, there is insufficient information to determine the preference. The action  $a_i$  is optimal, if there is no other action that is preferred to  $a_i$ .

In Section 4, we introduce the reverse logistics and its importance in the process of design for supply chain, before we demonstrate decision support based on the proposed robust BBN. The example of spacecraft circuit board recovery in reverse logistics is given in Section 5.

#### 4 Decision support in design for closed-loop supply chain

The IDSS is general and can be applied in different phases of product development. In particular, design for supply chain is one of the under-studied research areas in collaborative design and manufacturing. The objective of design for supply chain is to allow engineers to consider lifecycle costs of products from production, distribution, maintenance, to recycle during decision-makings at the product design phases. Engineers should make sound decisions in selecting product platforms, configurations and design parameters so that the costs associated with production, transportation from multi-tiered suppliers to OEMs, disassembly and recycling processes and remanufacturing channels from product consumers to OEMs.

Traditionally, the study of logistics management focuses on the forward supply chain, which is the delivery of products from manufacturer to marketplace. Only limited attention has been given to the reverse logistics, which is the flow of returning products from consumer to producer. The Council of Logistics Management published the first known definition of reverse logistics in the early 1990s, as ‘the role of logistics in recycling, waste disposal and management of hazardous materials; a broader perspective includes all related logistics activities carried out in source reduction, recycling, substitution, reuse of materials and disposal’ (Stock, 1992). The driving force for reverse logistics has been classified into three subgroups: economics, legislation and extended responsibilities. It has been realised that the total value of products returned in the US is estimated at \$100 billion annually (Stock et al., 2002).

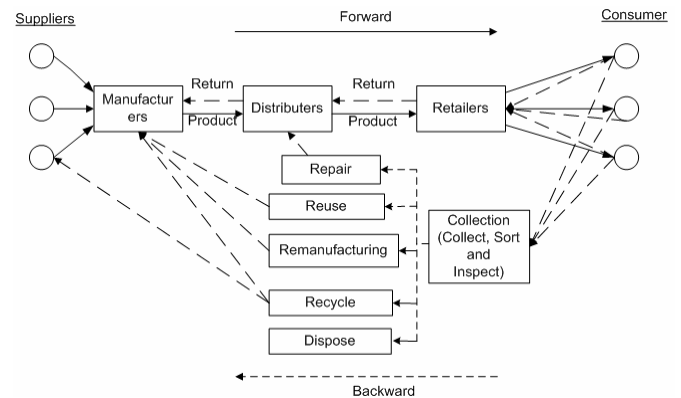
#### 4.1 Reverse logistics

Reverse logistics consists of planning, implementing and controlling the reverse flow of materials and management of related downstream information through the supply chain with the primary purpose of recapturing value. Thus, the associated decisions may drive a large extent of development in the process of manufacturing and remanufacturing, forward and backward material flows and related operational functions (Carter and Ellram, 1998).

Reverse logistics strategies for end-of-life products are usually developed to allow manufacturers to determine the optimal amount to spend on buy-back and the optimal unit cost of reverse logistics (Knemeyer et al., 2002). A good management strategy is to find the best choices of material recovery channels based on the conditions and values of the used products to maximise the recoverable residual values. Four major recovery choices are:

- **Reuse:** It is the process by which products are reused directly without prior operations. It may need cleaning and minor maintenance (e.g., reusable packages such as bottles, pallets or containers).
- **Repair:** It is the process of fixing or restoring failed products. However, there is a possibility of quality loss (e.g., industrial machines and electronic equipment).
- **Recycling:** It is the process of material recovery (e.g., scrap, glass, paper and plastic recycling).
- **Remanufacturing:** It is the process of disassembly and recovery of worn, defective or discarded products. Disassembled products and all components are cleaned and inspected. Those components which can be reused are brought to inspection and those that cannot be reused are replaced. A remanufactured product should match the same customer expectation as new products (e.g., mechanical assemblies such as aircraft engines and copy machines).

**Figure 1** General supply chain framework



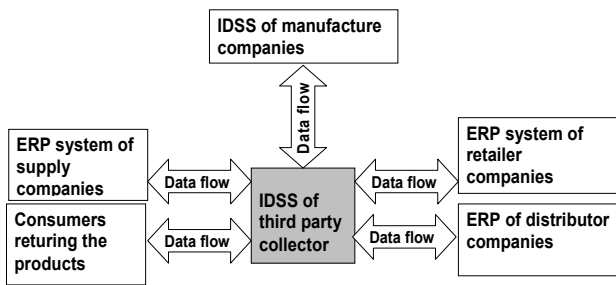
Source: Hamza et al. (2007)

We model the closed-loop supply chain with a general framework of the forward and reverse material flows in PLM. This framework includes the major scenarios that can take place in the recovering of the used product, as shown in Figure 1. In the figure, collection refers to all activities of rendering used products and physically transporting them for further treatment. Sorting and inspection are the operations that determine whether a given product is reusable and which method to apply. Thus, sort and inspect result in splitting the flow of used products according to the distinct types of recovery channels such as repair, reuse, remanufacturing and recycle.

#### 4.2 Decision-making in reverse logistics

To support a set of decision-makers working together as a group, a collaborative IDSS has some special technological requirements of hardware, software and procedures (Sean, 2001). Collaborative IDSS software also needs special functional capabilities, in addition to the capabilities of single user IDSS software, such as anonymous input of the user's ideas, listing group members' ideas, voting and ranking of the decision alternatives. IDSS will have the ability to take the integrated data stored within the database and transform them through various analysis techniques. ERP systems are able to achieve integration by bringing together data from different sources within the corporation. This may include disparate databases that exist across different functional units, thus, helping the firm to gain a more complete and realistic picture of all the data they hold. ERP systems have traditionally not been able to provide satisfactory support for transforming data and enabling decision-makers to discover and learn, ultimately turning this data into knowledge. This is where IDSS can give strong support. The human component of group IDSS should include a group facilitator, who leads the session by serving as the interface between the group and the computer systems (Shevtshenko, 2007).

**Figure 2** IDSS system used for reverse logistics in the collaborative supply chain network



As shown in Figure 2, the IDSS system can be applied in the collaborative supply chain networks to assist the third-party collector in decision-making. First of all, the data about the returned product are inserted into the IDSS system. The data about returns will be transmitted to the ERP systems of participants. Within the ERP systems, it will be possible to track information about returned products during the whole life cycle, until the product is disposed. It enables the participants to be prepared for the situation when the product should be repaired, remanufactured or reused. Based on the historical data, the participants are able to estimate the probability that the product will be disposed or the new products should be produced. This information will be used as evidences to support the decision-making. The users of the future IDSS systems can be business executives or some other groups of management (knowledge) workers.

One of the challenges for decision-making in reverse logistics management is the lack of information and knowledge (e.g., under which working environment products were used by customers, how they were maintained, what the long-term impact on environment and energy consumption will be) (Tibben-Limbke, 1998). Therefore, risks associated with environment, health, reusability, total cost of ownership, etc., should be considered (Thierry et al., 1995). The amount of data related to returned products is much less than that of new products flowing forward in the supply chain, since, the initiator of reverse logistics usually is end users, who are most likely to have no motivation to keep and share detailed product lifecycle information. Uncertainties are likely associated with information such as the reliability of reverse material flows, the quality and condition of returned products, the timing of returns, the potential residual values and the demand of the secondary market (Tan and Kumar, 2006). Moreover, there are uncertainties which arise from limited availability of data and deteriorated quality of data. Therefore, reverse logistics is characterised by much higher uncertain factors compared to regular forward material flows in supply chains. An appropriate representation of the uncertainties in reverse logistics is important.

Considering the high uncertainties, we can apply our robust Bayesian networks described in Section 3 to the selection of proper recovery channels and activities for returned products. The IDSS enables participants to be prepared for situations when products are ready to be returned and decisions of recovery channel selection need to

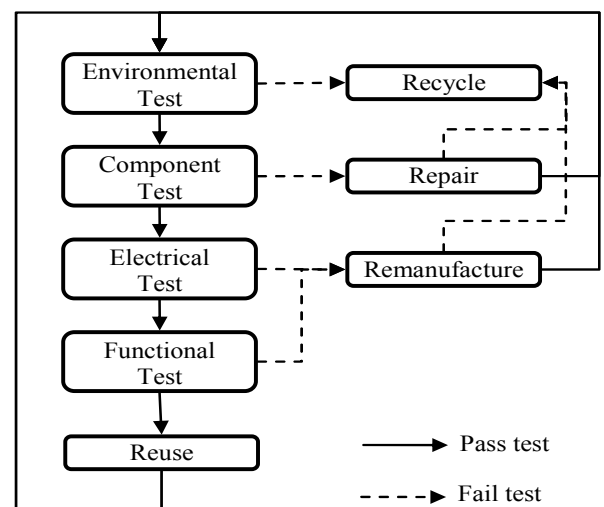
be made. The probabilities that products are repaired, remanufactured, reused or disposed can be used to support decision-making.

The available information to support decision-making for returned products is usually scarce. For this reason, the BBN mechanism with probabilistic reasoning is a good option in IDSS systems in reverse logistics management. BBN can quantitatively evaluate different options and propose what the best action could be. If the result of the previous action is known, this piece of information can in turn be used as extra evidence to update the probabilities for further estimations with increased accuracy. In Section 5, the new robust probabilistic reasoning approach is illustrated with an example of spacecraft circuit board recovery. The GBR theory is applied to monitor the probability of design project being successful and the comparison of different actions is presented.

## 5 Application of the robust BBNs to circuit board recovery

We apply the robust decision-making approach to an example of spacecraft circuit board recovery. Space systems are inherently risky because of the technology involved and the complexity of their activities. The significant presence of uncertainties requires good management of risks during the development of space systems. For example, space shuttle is recognised as the world's first reusable space transportation system. The values of components are continuously recovered and recaptured. NASA thereof is regarded as one of the major reverse logistics practitioners. Capturing of the uncertain conditions of these reusable components is critical in order to prevent disasters in PLM. Since, all phases in the spacecraft life cycle are associated with risks, development of a robust tool to calculate the accumulated cost and assess risks is essential in this industry.

**Figure 3** State diagram of performed tests and activities



When previously used circuit boards arrive at the reverse logistics collector, their conditions will be tested. The robust BBN mechanism can help the collector to decide what action should be performed to the recovered circuit boards. After several tests, appropriate actions should be selected. As shown in Figure 3, four tests are typically performed: environmental test; component test; electrical test and functional test. If the result of any test is negative, the appropriate action including recycle, repair, remanufacture and reuse will be performed. After the completion of any action, the lower and upper bounds of project success probability are updated. The decision-maker will monitor the posterior probabilities after each action, until the probability of project success is high enough.

The decisions must be made with the noticeable presence of uncertainty. Not enough prior information is available, since, the number of previous tests is small. We would like to estimate the probability that a project will be successful if we take an action of reuse, remanufacture or repair for some circuit boards, i.e.,  $P(Proj.Succ. = Yes | Reuse; Remanuf.; Repair)$ . As the prior probabilities and likelihood functions are given as intervals, the posterior probabilities will also be intervals.

We calculate the range of the project success probability. The narrower is the range, the lesser is the

indeterminacy our decision will have. The network is built on the base of prior information. The intervals of project success rate can be calculated according to equation (3) and equation (4).

A simple Bayesian network example consisting of three nodes is used here to introduce how robust Bayesian networks are constructed, then, later in this section, a more comprehensive network is used for further calculations. As shown in Figure 4, relationships of environmental test, component test and project success are built. Environmental test is related to project success and component test is related to both environmental test and project success. The lower and upper bounds of prior probabilities and likelihood functions are given in Table 1.

Figure 4 The BBN model of simple tests for project

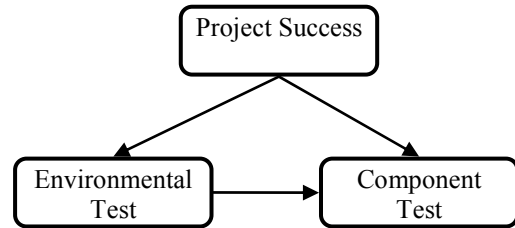


Table 1 Interval prior probabilities and likelihood probabilities (where *lb* denotes lower bound and *ub* denotes upper bound) for the Bayesian network in Figure 4

Prior probabilities and likelihood functions	lb	ub
$P(\text{ProjectSuccess} = \text{Yes})$	0.90	0.95
$P(\text{ProjectSuccess} = \text{No})$	0.05	0.10
$P(\text{Envir. Test} = \text{OK}   \text{ProjectSuccess} = \text{Yes})$	0.97	0.99
$P(\text{Envir. Test} = \text{Failed}   \text{ProjectSuccess} = \text{Yes})$	0.01	0.03
$P(\text{Envir. Test} = \text{OK}   \text{ProjectSuccess} = \text{No})$	0.75	0.8
$P(\text{Envir. Test} = \text{Failed}   \text{ProjectSuccess} = \text{No})$	0.2	0.25
$P(\text{Comp. Test} = \text{OK}   \text{ProjectSuccess} = \text{No})$	0.6	0.7
$P(\text{Comp. Test} = \text{Failed}   \text{ProjectSuccess} = \text{No})$	0.3	0.4
$P(\text{Comp. Test} = \text{OK}   \text{ProjectSuccess} = \text{Yes}; \text{Envir. Test} = \text{OK})$	0.98	0.995
$P(\text{Comp. Test} = \text{Failed}   \text{ProjectSuccess} = \text{Yes}; \text{Envir. Test} = \text{OK})$	0.005	0.02
$P(\text{Comp. Test} = \text{OK}   \text{ProjectSuccess} = \text{No}; \text{Envir. Test} = \text{OK})$	0.6	0.7
$P(\text{Comp. Test} = \text{Failed}   \text{ProjectSuccess} = \text{No}; \text{Envir. Test} = \text{OK})$	0.3	0.4

To calculate the two posterior probabilities

$$\underline{P}(\text{Proj.Succ.} = \text{Yes} | \text{Env.Test} = \text{OK}; \text{Comp.Test} = \text{OK})$$

and

$$\bar{P}(\text{Proj.Succ.} = \text{No} | \text{Env.Test} = \text{OK}; \text{Comp.Test} = \text{OK})$$

in the lower bound network, we use the prior probabilities and likelihood bounds listed in Table 2.

Table 2 Prior probabilities and likelihood functions for the nodes in the lower bound Bayesian network

		Proj.Succ. = Yes	Proj.Succ. = No
Prior probability of project success		0.90 (lb)	0.10 (ub)
Environmental test node probabilities (test 1)	OK1	0.97 (lb)	0.8 (ub)
	Failed1	0.03 (ub)	0.2 (lb)
Component test node probabilities (test 2)	OK2	0.98 (lb)	0.7 (ub)
	Failed2	0.02 (ub)	0.3 (lb)

Based on equation (6), the lower bound of the posterior probability that ‘project is successful, given that the environmental test is OK and component test is OK’ can be calculated as

$$\begin{aligned} & \underline{P}(\text{Proj.Succ.} = \text{Yes} \mid \text{Env.Test} = \text{OK}; \text{Comp.Test} = \text{OK}) \\ &= \frac{\left[ \begin{array}{l} \underline{P}(\text{Env.Test} = \text{OK} \mid \text{Proj.Succ.} = \text{Yes}) \cdot \\ \underline{P}(\text{Comp.Test} = \text{OK} \mid \text{Proj.Succ.} = \text{Yes}; \text{Env.Test} = \text{OK}) \cdot \\ \underline{P}(\text{Proj.Succ.} = \text{Yes}) \end{array} \right]}{\left[ \begin{array}{l} \underline{P}(\text{Env.Test} = \text{OK} \mid \text{Proj.Succ.} = \text{Yes}) \cdot \\ \underline{P}(\text{Comp.Test} = \text{OK} \mid \text{Proj.Succ.} = \text{Yes}; \text{Env.Test} = \text{OK}) \cdot \\ \underline{P}(\text{Proj.Succ.} = \text{Yes}) \\ + \underline{P}(\text{Env.Test} = \text{OK} \mid \text{Proj.Succ.} = \text{No}) \cdot \\ \underline{P}(\text{Comp.Test} = \text{OK} \mid \text{Proj.Succ.} = \text{No}; \text{Env.Test} = \text{OK}) \cdot \\ \underline{P}(\text{Proj.Succ.} = \text{No}) \end{array} \right]} \\ &= \frac{0.97 \times 0.98 \times 0.9}{(0.97 \times 0.98 \times 0.9) + (0.8 \times 0.7 \times 0.1)} = 0.938565505 \end{aligned}$$

Based on equation (7), the upper bound of the posterior probability that ‘project is not successful, given that the environmental test is OK and component test is OK’ can be calculated as

$$\begin{aligned} & \overline{P}(\text{Proj.Succ.} = \text{No} \mid \text{Env.Test} = \text{OK}; \text{Comp.Test} = \text{OK}) \\ &= \frac{\left[ \begin{array}{l} \overline{P}(\text{Env.Test} = \text{OK} \mid \text{Proj.Succ.} = \text{No}) \cdot \\ \overline{P}(\text{Comp.Test} = \text{OK} \mid \text{Proj.Succ.} = \text{No}; \text{Env.Test} = \text{OK}) \cdot \\ \overline{P}(\text{Proj.Succ.} = \text{No}) \end{array} \right]}{\left[ \begin{array}{l} \overline{P}(\text{Env.Test} = \text{OK} \mid \text{Proj.Succ.} = \text{No}) \cdot \\ \overline{P}(\text{Comp.Test} = \text{OK} \mid \text{Proj.Succ.} = \text{No}; \text{Env.Test} = \text{OK}) \cdot \\ \overline{P}(\text{Proj.Succ.} = \text{No}) \\ + \underline{P}(\text{Env.Test} = \text{OK} \mid \text{Proj.Succ.} = \text{Yes}) \cdot \\ \underline{P}(\text{Comp.Test} = \text{OK} \mid \text{Proj.Succ.} = \text{Yes}; \text{Env.Test} = \text{OK}) \cdot \\ \underline{P}(\text{Proj.Succ.} = \text{Yes}) \end{array} \right]} \\ &= \frac{0.8 \times 0.7 \times 0.1}{(0.8 \times 0.7 \times 0.1) + (0.97 \times 0.98 \times 0.9)} = 0.061434495 \end{aligned}$$

Similarly, to calculate the posterior probabilities

$$\overline{P}(\text{Proj.Succ.} = \text{Yes} \mid \text{Env.Test} = \text{OK}; \text{Comp.Test} = \text{OK})$$

and

$$\underline{P}(\text{Proj.Succ.} = \text{No} \mid \text{Env.Test} = \text{OK}; \text{Comp.Test} = \text{OK})$$

in the upper bound network, we use the prior probabilities and likelihood bounds listed in Table 3.

Based on equation (7), the upper bound of ‘project is successful, given that the environmental test is OK and component test is OK’ is calculated as

$$\begin{aligned} & \overline{P}(\text{Proj.Succ.} = \text{Yes} \mid \text{Env.Test} = \text{OK}; \text{Comp.Test} = \text{OK}) \\ &= \frac{\left[ \begin{array}{l} \overline{P}(\text{Env.Test} = \text{OK} \mid \text{Proj.Succ.} = \text{Yes}) \cdot \\ \overline{P}(\text{Comp.Test} = \text{OK} \mid \text{Proj.Succ.} = \text{Yes}; \text{Env.Test} = \text{OK}) \cdot \\ \overline{P}(\text{Proj.Succ.} = \text{Yes}) \end{array} \right]}{\left[ \begin{array}{l} \overline{P}(\text{Env.Test} = \text{OK} \mid \text{Proj.Succ.} = \text{Yes}) \cdot \\ \overline{P}(\text{Comp.Test} = \text{OK} \mid \text{Proj.Succ.} = \text{Yes}; \text{Env.Test} = \text{OK}) \cdot \\ \overline{P}(\text{Proj.Succ.} = \text{Yes}) \\ + \underline{P}(\text{Env.Test} = \text{OK} \mid \text{Proj.Succ.} = \text{No}) \cdot \\ \underline{P}(\text{Comp.Test} = \text{OK} \mid \text{Proj.Succ.} = \text{No}; \text{Env.Test} = \text{OK}) \cdot \\ \underline{P}(\text{Proj.Succ.} = \text{No}) \end{array} \right]} \\ &= \frac{0.99 \times 0.995 \times 0.95}{(0.99 \times 0.995 \times 0.95) + (0.75 \times 0.6 \times 0.05)} = 0.976520861 \end{aligned}$$

**Table 3** Prior probabilities and likelihood functions for the nodes in the upper bound Bayesian network

		Proj.Succ. = Yes	Proj.Succ. = No
Prior probability of	project success	0.95 (ub)	0.05 (lb)
Environmental test node probabilities	OK1	0.99 (ub)	0.75 (lb)
	Failed1 (test 1)	0.01 (lb)	0.25 (ub)
Component test node probabilities	OK2	0.995 (ub)	0.6 (lb)
	Failed2 (test 2)	0.005 (lb)	0.4 (ub)

Based on equation (6), the lower bound of ‘project is not successful, given that the environmental test is OK and component test is OK’ is calculated as

$$\begin{aligned} & \underline{P}(\text{Proj.Succ.} = \text{No} \mid \text{Env.Test} = \text{OK}; \text{Comp.Test} = \text{OK}) \\ &= \frac{\left[ \begin{array}{l} \underline{P}(\text{Env.Test} = \text{OK} \mid \text{Proj.Succ.} = \text{No}) \cdot \\ \underline{P}(\text{Comp.Test} = \text{OK} \mid \text{Proj.Succ.} = \text{No}; \text{Env.Test} = \text{OK}) \cdot \\ \underline{P}(\text{Proj.Succ.} = \text{No}) \end{array} \right]}{\left[ \begin{array}{l} \underline{P}(\text{Env.Test} = \text{OK} \mid \text{Proj.Succ.} = \text{No}) \cdot \\ \underline{P}(\text{Comp.Test} = \text{OK} \mid \text{Proj.Succ.} = \text{No}; \text{Env.Test} = \text{OK}) \cdot \\ \underline{P}(\text{Proj.Succ.} = \text{No}) \\ + \overline{P}(\text{Env.Test} = \text{OK} \mid \text{Proj.Succ.} = \text{Yes}) \cdot \\ \overline{P}(\text{Comp.Test} = \text{OK} \mid \text{Proj.Succ.} = \text{Yes}; \text{Env.Test} = \text{OK}) \cdot \\ \overline{P}(\text{Proj.Succ.} = \text{Yes}) \end{array} \right]} \\ &= \frac{0.75 \times 0.6 \times 0.05}{(0.75 \times 0.6 \times 0.05) + (0.99 \times 0.995 \times 0.95)} = 0.023479139 \end{aligned}$$

Therefore, the interval posterior probabilities are

$$\begin{aligned} & P(\text{Proj.Succ.} = \text{Yes} \mid \text{Env.Test} = \text{OK}; \text{Comp.Test} = \text{OK}) \\ &= [0.938565505, 0.976520861] \end{aligned}$$

and

$$P(\text{Proj.Succ.} = \text{No} \mid \text{Env.Test} = \text{OK}; \text{Comp.Test} = \text{OK}) \\ = [0.023479139, 0.061434495]$$

Notice that

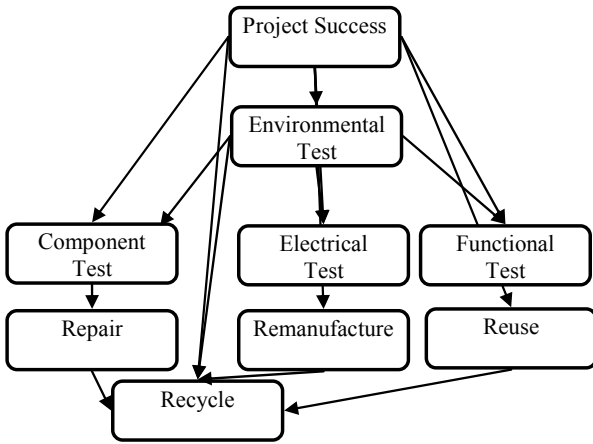
$$\bar{P}(\text{Proj.Succ.} = \text{No} \mid \text{Env.Test} = \text{OK}; \text{Comp.Test} = \text{OK}) \\ = 1 - \underline{P}(\text{Proj.Succ.} = \text{Yes} \mid \text{Env.Test} = \text{OK}; \text{Comp.Test} = \text{OK})$$

which satisfies the logic coherence constraint.

If the goal of conducting the tests is to select a strategy of board recovery so that the probability of project success should be greater than 0.99, then environmental test and component test are not enough. More tests are required.

Now, we move to the more comprehensive example, where all tests are considered. Figure 5 shows the BBN model containing all tests.

**Figure 5** The BBN model for project with all tests included



Suppose, we would like to decide what tests should be performed in order to have a project successful rate of at least 99%. The decision support process by the robust BBN is illustrated step by step as follows.

Step 1 After a board passes the environmental test, we receive the posterior probability  
 $P(\text{Proj.Succ.} = \text{Yes} \mid \text{Env.Test} = \text{OK}) = [0.916, 0.958]$ .

The probabilities for different actions will also be updated as:

$$P(\text{Repair} = \text{Yes} \mid \text{Env.Test} = \text{OK}) = [0.0172, 0.0475];$$

$$P(\text{Remanuf.} = \text{Yes} \mid \text{Env.Test} = \text{OK}) = [0.0051, 0.0187];$$

$$P(\text{Reuse} = \text{Yes} \mid \text{Env.Test} = \text{OK}) = [0.874, 0.913];$$

$$P(\text{Recycle} = \text{Yes} \mid \text{Env.Test} = \text{OK}) = [0.0894, 0.13].$$

The robust BBN tool is also able to estimate the posterior probability of project success after different actions are successfully performed as:

$$P(\text{Proj.Succ.} = \text{Yes} \mid \text{Env.Test} = \text{OK}; \text{Repaired} = \text{Yes}) \\ = [0.275, 0.382]$$

$$P(\text{Proj.Succ.} = \text{Yes} \mid \text{Env.Test} = \text{OK}; \text{Remanuf.} = \text{Yes}) \\ = [0.959, 0.969];$$

$$P(\text{Proj.Succ.} = \text{Yes} \mid \text{Env.Test} = \text{OK}; \text{Reuse} = \text{Yes}) \\ = [0.995, 0.998].$$

It can be seen that, the most preferred action to take is to reuse the board since it has the highest probability lower bound. The probability  $P(\text{Proj.Succ.} = \text{Yes} \mid \text{Env.Test} = \text{OK}) = [0.916, 0.958]$  is not high enough to reach the target of 0.99. So we need to perform the further component test.

Step 2 If the board passes the component test, we receive the posterior probability

$$P(\text{Proj.Succ} = \text{Yes} \mid \text{Env.Test} = \text{OK}; \text{CompTest} = \text{OK}) \\ = [0.973, 0.983]$$

The probabilities for different actions will be also updated as:

$$P(\text{Repair} = \text{Yes} \mid \text{Env.Test} = \text{OK}; \text{Comp.Test} = \text{OK}) = [0]$$

$$P(\text{Remanuf} = \text{Yes} \mid \text{Env.Test} = \text{OK}; \text{Comp.Test} = \text{OK}) \\ = [0.0051, 0.0195]$$

$$P(\text{Reuse} = \text{Yes} \mid \text{Env.Test} = \text{OK}; \text{Comp.Test} = \text{OK}) \\ = [0.925, 0.935]$$

$$P(\text{Recycle} = \text{Yes} \mid \text{Env.Test} = \text{OK}; \text{Comp.Test} = \text{OK}) \\ = [0.0663, 0.0761].$$

The posterior probabilities of project being successful after taking different actions are estimated as

$$P(\text{Proj.Succ.} = \text{Yes} \mid \text{Env.Test} = \text{OK}; \\ \text{Comp.Test} = \text{OK}; \text{Repaired} = \text{Yes}) = \text{not\_valid}$$

$$P(\text{Proj.Succ.} = \text{Yes} \mid \text{Env.Test} = \text{OK}; \\ \text{Comp.Test} = \text{OK}; \text{Remanuf} = \text{Yes}) = [0.979, 0.992]$$

$$P(\text{Proj.Succ.} = \text{Yes} \mid \text{Env.Test} = \text{OK}; \\ \text{Comp.Test} = \text{OK}; \text{Reuse} = \text{Yes}) = [0.9908, 0.9975];$$

It means that after a board passes the component test, the repair action should not be taken anymore.

Since the probability

$$P(\text{Proj.Succ.} = \text{Yes} \mid \text{Env.Test} = \text{OK}; \text{Comp.Test} = \text{OK}) \\ = [0.973, 0.983]$$

has not reached the required 0.99, electrical test should be performed.

Step 3 If the board passes the electrical test, we receive the posterior probability

$$P(\text{Proj.Succ.} = \text{Yes} \mid \text{Env.Test} = \text{OK}; \\ \text{Comp.Test} = \text{OK}; \text{Electr.Test} = \text{OK}) = [0.991, 0.993]$$

The probabilities for different actions are also updated as:

$$P(\text{Repair} = \text{Yes} \mid \text{Env.Test} = \text{OK};$$

$$\text{Comp.Test} = \text{OK}; \text{Electr.Test} = \text{OK}) = [0]$$

$$P(\text{Remanuf.} = \text{Yes} \mid \text{Env.Test} = \text{OK};$$

$$\text{Comp.Test} = \text{OK}; \text{Electr.Test} = \text{OK}) = [0]$$

$$P(\text{Reuse} = \text{Yes} \mid \text{Env.Test} = \text{OK};$$

$$\text{Comp.Test} = \text{OK}; \text{Electr.Test} = \text{OK}) = [0.942, 0.944]$$

$$P(\text{Recycle} = \text{Yes} \mid \text{Env.Test} = \text{OK};$$

$$\text{Comp.Test} = \text{OK}; \text{Electr.Test} = \text{OK}) = [0.0566, 0.0581]$$

The posterior probabilities of project being successful after different actions are taken are estimated as:

$$P(\text{Proj.Succ.} = \text{Yes} \mid \text{Env.Test} = \text{OK}; \text{Comp.Test} = \text{OK};$$

$$\text{Electr.Test} = \text{OK}; \text{Repaired} = \text{Yes}) = \text{not\_valid}$$

$$P(\text{Proj.Succ.} = \text{Yes} \mid \text{Env.Test} = \text{OK}; \text{Comp.Test} = \text{OK};$$

$$\text{Electr.Test} = \text{OK}; \text{Remanuf} = \text{Yes}) = \text{not\_valid}$$

$$P(\text{Proj.Succ.} = \text{Yes} \mid \text{Env.Test} = \text{OK}; \text{Comp.Test} = \text{OK};$$

$$\text{Reuse} = \text{Yes}) = [0.99955, 0.99963]$$

When the electrical test is passed, the posterior probability of project success becomes  $P(\text{Proj.Succ.} = \text{Yes} \mid \text{Env.Test} = \text{OK}; \text{Comp.Test} = \text{OK}; \text{Electr.Test} = \text{OK}) = [0.992, 0.995]$

The probability of project being successful is high enough to meet the minimal requirement. The result suggests that the further functional test is not necessary.

As illustrated by the above procedure, the robust BBN tool monitors the upper and lower bounds of posterior probabilities until the lower bound of project success probability is high enough. It is also possible to predict the probability of success after every action and decide what should be made next. This allows us to increase the effectiveness and robustness of decision-making.

In summary, the robust BBN mechanism considers the latest available evidence. It recalculates and updates posterior data after new information is available. The uncertainties can be monitored and used to estimate the worst case scenario. The imprecise probabilities can increase the confidence of decision-makers compared to the traditional precise probabilistic reasoning. While, probabilistic distributions provide more information than deterministic estimations, interval probabilities can provide even more information than precise probabilities.

## 6 Concluding remarks

In this paper, we presented a robust probabilistic reasoning framework based on imprecise probabilities for robust decision support. This model explicitly differentiates uncertainty from variability and incorporates uncertainty factors due to lack of perfect knowledge. Interval probabilities are used to represent classes of possible variations instead of precise ones, which captures

imprecision and indeterminacy. This allows us to consider all possible scenarios between extreme cases during probabilistic reasoning. A GBR was developed based on interval probabilities such that reliable decision-making can be supported. This robust decision support tool is useful in PLM in collaborative networks, where lack of data and information increases risks of decision-making. The new approach can be used to improve the robustness of decisions under high uncertainties in IDSSs.

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