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Semantic Tolerancing

Yan Wang
Department of Industrial Engineering and Management Systems
University of Central Florida
Orlando, FL 32816, USA

Abstract

Recently we developed a new tolerance modeling scheme, semantic tolerance modeling, to enable interpretable and accurate tolerance analysis. In this paper, a new dimensioning and tolerancing practice, semantic tolerancing, is proposed with the theoretical support of semantic tolerance modeling. Based on the principles of interpretability, this tolerancing approach allows for better design intent capturing, including flexible material selection, component sorting in selective assembly, and assembly sequence.

Keywords
Semantic tolerance modeling, design intent, modal interval analysis, semantic tolerancing

1. Introduction

Tolerance modeling forms an important link between design and manufacturing. A significant amount of research has been given to explore the mathematical basis for 3D dimensional and geometric tolerance representation, analysis, and synthesis. Relations among tolerances in components and assemblies are formulated in different ways and solved numerically. The typical analysis methods include variational estimation, kinematic formulation, statistical approximation, and Monte Carlo simulation. However, current tolerance modeling methods do not represent the semantics of tolerance specifications well.

First, traditional tolerance analysis methods assume objects all have rigid geometry. Variance is increasingly stacked up as components are assembled. Geometric variation of assembly is always assumed to be larger than those of its subassemblies and components. This rigid body tolerance analysis overestimates tolerance accumulations of compliant assemblies with flexible materials, such as assemblies containing sheet metal and plastic parts, which are common in aerospace, automobile, and electronics industry.

Second, current modeling and analysis methods do not maintain the semantics of tolerance specifications during model formulation and numerical computing. These specifications, along with relations among them, imply manufacturing and assembly methods, especially the sequence of fabrication as an important component of design intent. Tolerance analysis is usually simplified to the computation of numerical intervals. However, logical dependency and algebraic relations among variations are left out in existing approaches. This leads to the problem that numerical solutions are not interpretable. Instead of focusing only on mathematical and numerical convenience, a good model of tolerance should convey the full semantics of size and geometric tolerances and support analysis and synthesis with a simple yet comprehensive structure.

Semantic tolerance modeling [1, 2, 3] is a recently proposed tolerance analysis model to better capture tolerancing intent. It is to embed logical meanings and engineering implications into the mathematical representation based on modal interval analysis [4]. Numerical results are interpretable based on certain interpretability principles. In this paper, we introduce a new dimensioning and tolerancing concept, semantic tolerancing, for engineering practice. Extended from traditional set-based interval, modal interval introduces logical quantifiers and provides interpretation of intervals. In the remainder of the paper, Section 2 compares modal intervals and traditional intervals and gives an overview of semantic tolerance modeling. Section 3 describes the semantic representation for specification priority, assembly sequence, and material properties. Section 4 presents the concept of semantic tolerancing with new range symbols.

2. Semantic Tolerance Modeling

2.1 Modal Interval Analysis vs. Interval Analysis

Interval mathematics [5] is a generalization in which interval numbers replace real numbers, interval arithmetic replaces real arithmetic, and interval analysis replaces real analysis. The set of intervals corresponding to real numbers is

$$\mathbb{IR} = \{[a,b] \mid a \in \mathbb{R}, b \in \mathbb{R}\} \quad (1)$$

Let $[a]=[\underline{a},\overline{a}]$, $[b]=[\underline{b},\overline{b}]$ be real intervals and $\circ$ be one of the four basic arithmetic operations for real numbers, $\circ \in \{+,-,\cdot,/\}$. The corresponding operations for interval $[a]$ and $[b]$ are defined by

$$[a]\circ[b] = \{x \circ y \mid x \in [a], y \in [b]\} \quad (2)$$

Not only intervals solve the problem of representation for real numbers on a digital scale, but they are the most suitable way to represent uncertainties and errors in technical constructions, measuring, computations, and ranges of fluctuation and variation. Interval analysis has been extensively used in reliable computing in computer science. In engineering fields, methods of interval analysis have been used in robust geometry construction and evaluation [6], set-based modeling [7], imprecise structural analysis [8], design optimization [9], finite-element formulation and analysis [10], soft constraint solving [11].

Modal interval analysis (MIA) [4, 12, 13, 14, 15] is a logical and semantic extension of classical interval analysis (IA). Unlike the classical IA which identifies an interval by a set of real numbers as in Eqn. (1), MIA identifies an interval by a set of predicates which is fulfilled by the real numbers. The modal quantifier associates to every real predicate. A real function $f(x)$ where $x \in \mathbb{R}^n$ can be extended to $f(\mathbf{x})$ where $\mathbf{x} \in \mathbb{KR}^n$, which is called $\mathbb{KR}$-extension or AE-extension. In special cases of the real arithmetic operations, i.e., $f(x,y) = x \circ y$ with $\circ \in \{+,-,\times,\div\}$, the $\mathbb{KR}$-extensions lead to the so-called Kaucher arithmetic [16].

Given a set of closed intervals of real numbers in $\mathbb{R}$, the set of logical existential ($\exists$) and universal quantifiers ($\forall$), each *modal interval* or *generalized interval* $\mathbf{x} := [\underline{x},\overline{x}] \in \mathbb{KR}$ has an associated quantifier. $\mathbf{x}$ is called *proper* interval when $\underline{x} \le \overline{x}$ and called *improper* interval when $\underline{x} \ge \overline{x}$. The set of proper intervals is denoted by $\mathbb{IR} = \{[\underline{x},\overline{x}] \mid \underline{x} \le \overline{x}\}$, and the set of improper interval is $\overline{\mathbb{IR}} = \{[\underline{x},\overline{x}] \mid \underline{x} \ge \overline{x}\}$.

Three special operators, *pro*, *imp*, and *dual*, are defined in Kaucher arithmetic. Given a generalized interval $\mathbf{x} = [\underline{x},\overline{x}] \in \mathbb{KR}$, $\mathrm{pro}\,\mathbf{x} := [\min(\underline{x},\overline{x}), \max(\underline{x},\overline{x})]$ and $\mathrm{imp}\,\mathbf{x} := [\max(\underline{x},\overline{x}), \min(\underline{x},\overline{x})]$ return the proper and improper interval values of $\mathbf{x}$ respectively. $\mathrm{dual}[\underline{x},\overline{x}] := [\overline{x},\underline{x}]$ builds a relationship between proper and improper intervals. Related to arithmetic operations, $(\mathrm{dual}\,\mathbf{x}) \circ (\mathrm{dual}\,\mathbf{y}) = \mathrm{dual}(\mathbf{x} \circ \mathbf{y})$. The *inclusion* relation between modal intervals is defined as $[\underline{x},\overline{x}] \subseteq [\underline{y},\overline{y}] \Leftrightarrow \underline{y} \le \underline{x} \wedge \overline{x} \le \overline{y}$. The *less or equal* relation is defined as $[\underline{x},\overline{x}] \le [\underline{y},\overline{y}] \Leftrightarrow \underline{x} \le \underline{y} \wedge \overline{x} \le \overline{y}$. Table 1 lists the major difference between MIA and IA.

Table 1: The major differences between MIA and traditional IA

	Classical Interval Analysis	**Modal Interval Analysis**
Validity	$[3,2]$ is an invalid or empty interval	Both $[2,3]$ and $[3,2]$ are valid intervals
Semantics richness	$[2,3]+[2,4]=[4,7]$ is the only valid relation for +, and it only means "stack-up" and "worst-case". $-,\times,/$ are similar.	$[2,3]+[2,4]=[4,7]$, $[2,3]+[4,2]=[6,5]$, $[3,2]+[2,4]=[5,6]$, $[3,2]+[4,2]=[7,4]$ are all valid, and each has a different meaning. $-,\times,/$ have similar semantic properties.
Completeness of arithmetic	$\mathbf{a}+\mathbf{x}=\mathbf{b}$, but $\mathbf{x} \ne \mathbf{b}-\mathbf{a}$ $[2,3]+[2,4]=[4,7]$, $[2,4] \ne [4,7]-[2,3]$ $\mathbf{a}\times\mathbf{x}=\mathbf{b}$, but $\mathbf{x} \ne \mathbf{b}/\mathbf{a}$ $[2,3]\times[3,4]=[6,12]$, $[3,4] \ne [6,12]/[2,3]$ $\mathbf{x}-\mathbf{x} \ne 0$ $[2,3]-[2,3]=[-1,1] \ne 0$	$\mathbf{a}+\mathbf{x}=\mathbf{b}$, and $\mathbf{x}=\mathbf{b}-\mathrm{dual}\,\mathbf{a}$ $[2,3]+[2,4]=[4,7]$, $[2,4]=[4,7]-[3,2]$ $\mathbf{a}\times\mathbf{x}=\mathbf{b}$, and $\mathbf{x}=\mathbf{b}/\mathrm{dual}\,\mathbf{a}$ $[2,3]\times[3,4]=[6,12]$, $[3,4]=[6,12]/[3,2]$ $\mathbf{x}-\mathrm{dual}\,\mathbf{x}=0$ $[2,3]-[3,2]=0$

2.2 Semantic Tolerance Modeling

MIA is able to model problems on a logic basis and to obtain the interval functional evaluations for the mathematical model involved. Based on modal intervals, we recently proposed a new semantic tolerance modeling scheme in which the implications of tolerance stacking can be embedded in tolerance models.

The purpose of semantic tolerance modeling is to capture logical therefore engineering meanings and implications in mathematical representation, which is to build a bridge between mathematical theory and engineering practice. Semantic tolerance modeling has two important characteristics: (1) Interpretability: being able to interpret tolerance intervals during analysis and synthesis processes and to provide the basic understanding of tolerancing semantics; and (2) Optimality: being able to analyze tolerance propagation and accumulation so that tolerances can be specified without losing the basic requirements of completeness and soundness. Interpretability allows tolerance semantics to be embedded in interval results. Optimality assures tightness of variation estimation.

3. Semantics Representation

The uniqueness of generalized intervals is the semantic extension of intervals with logic quantifiers. As a set of predicates, semantics of an interval $\mathbf{x} \in \mathbb{KR}$ is denoted by $(\mathrm{Q}_{\mathbf{x}} x \in \mathrm{pro}\,\mathbf{x})$ where $\mathrm{Q}_{\mathbf{x}} \in \{\exists, \forall\}$. An interval $\mathbf{x} \in \mathbb{KR}$ is called *existential* if $\mathrm{Q}_{\mathbf{x}} = \exists$. Otherwise, it is called *universal* if $\mathrm{Q}_{\mathbf{x}} = \forall$. If a real relation $z = f(x_1, \cdots, x_n)$ is extended to the interval relation $\mathbf{z} = \mathbf{f}(\mathbf{x}_1, \cdots, \mathbf{x}_n)$, the interval relation $\mathbf{z}$ is interpretable if there is a semantic relation

$$(Q_1 x_1 \in \mathrm{pro}\,\mathbf{x}_1)\ldots(Q_n x_n \in \mathrm{pro}\,\mathbf{x}_n)(\mathrm{Q}_{\mathbf{z}}\, z \in \mathbf{z})(z = f(x_1, \cdots, x_n)) \quad (3)$$

3.1 Interpretability

Let $f : \mathbb{R}^n \to \mathbb{R}$ be a rational continuous function. Its modal rational extension $\mathbf{f} : \mathbb{KR}^n \to \mathbb{KR}$ replaces the real variables of f with modal interval variables and real operators with interval operators. The semantics of a modal interval relation or function is embodied in the relation's syntax. The syntax of a function $f(x_1, \ldots, x_n) : \mathbb{R}^n \to \mathbb{R}$ can be represented by a syntax tree. A component x_i is uni-incident in the function $f(x_1, \ldots, x_n)$ if it occupies only one leaf of the syntax tree. Otherwise, it is multi-incident. Leaves and branches of the syntax tree are connected with either one-variable operators such as $|\,|$ and $\sqrt{\ }$, or two-variable operators such as $+, -, \times, /$.

For a modal rational function $\mathbf{f}(\mathbf{x}) : \mathbb{KR}^n \to \mathbb{KR}$, if all arguments of $\mathbf{f}(\mathbf{x})$ are uni-incident, it can be interpreted as

$$(\forall x_p \in \mathrm{pro}\,\mathbf{x}_p)(\mathrm{Q}_{\mathbf{f}}\, z \in \mathrm{pro}\,\mathbf{f}(\mathbf{x}))(\exists x_i \in \mathrm{pro}\,\mathbf{x}_i)(z = f(x_p, x_i)) \quad (4)$$

or

$$(\forall x_i \in \mathrm{pro}\,\mathbf{x}_i)(\mathrm{Q}_{\mathrm{dual}\,\mathbf{f}}\, z \in \mathrm{pro}\,\mathbf{f}(\mathbf{x}))(\exists x_p \in \mathrm{pro}\,\mathbf{x}_p)(z = f(x_p, x_i)) \quad (5)$$

where (x_p, x_i) is the component splitting corresponding to the interval vector $\mathbf{x} = (\mathbf{x}_p, \mathbf{x}_i)$, with sub-vectors $\mathbf{x}_p$ and $\mathbf{x}_i$ containing proper and improper components respectively.

Different semantics of linear tolerance stack-up in assembly enclosures needs to be differentiated in tolerance design. This includes the semantics associated with assembly sequence, accuracy of tolerance estimation, and controllability of variation. As tolerances are stacked up in a tolerance chain, the earlier a part is assembled in the sequential process, the less controllable the corresponding variations are in order to close the chain. In an assembly line, correction or adjustment of individual components in a previously finished subassembly, especially those directly from suppliers, to meet functional requirement is more difficult than the adjustment of newly added ones. In this sense, tolerances of earlier assembled parts are out of the control of current worker. They are *uncontrollable* tolerances. In contrast, the most recently assembled ones have *controllable* tolerances.

If a dimension is functionally critical and therefore explicitly specified in design and blueprint, it is called working dimension. On the other hand, a dimension is called balance or reference dimension if it is not explicitly specified and its nominal value and tolerance are calculated from working dimensions. Compared to working dimensions, which are hard requirements imposed a priori, balance dimensions are soft and derived a posteriori. In general, *a priori tolerances* are tolerances with predetermined variations, while *a posteriori tolerances* are tolerances with derived variations. Whenever defining a relation among tolerances, we have implicitly differentiated these two types of tolerances.

Based on manufacturing and assembly sequences, tolerances may be specified in different ways to designate desirable semantics. For example, in Figure 1, dimensions a, b, and c in three components have the relation

$a + b = c$. If Part A and B are provided by suppliers and Part C is to be built in house (Figure 1-a, Case I), or if a and b are working dimensions and c is a balance dimension, the tolerance of c is determined by the tolerances of a and b, and the tolerance chain should be closed. In this case, the semantics of "given A and B, C needs to fit A and B" is expressed as $(\forall a \in \mathrm{pro}\,\mathbf{a})(\forall b \in \mathrm{pro}\,\mathbf{b})(\exists c \in \mathrm{pro}\,\mathbf{c})(a + b = c)$, which is different from the semantics of "given A, B and C need to fit A" when a is a working dimension while b and c are balance dimensions (Figure 1-b, Case II).

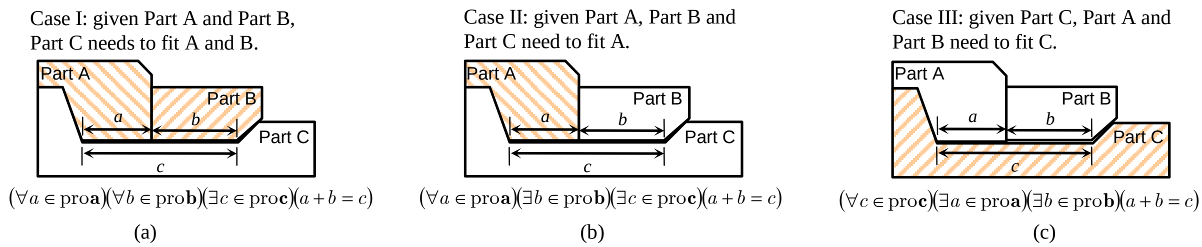

Figure 1: Different semantics needs to be captured, which are not differentiated in traditional modeling methods

In semantic tolerance models, a priori and a posteriori tolerances are differentiated by modalities of intervals. A priori tolerance has the semantics of uncontrollable, unchangeable, critical, hard-constrained, specified, etc. For example, tolerances in working dimensions are categorized as a priori. A posteriori tolerance has the semantics of flexible, soft-constrained, adjustable, controllable, feedback, etc. A posteriori variations provide "buffers" in tolerance allocation to make algebraic relations valid and close the tolerance chain, such as balance dimensions. Notice that the categorization of a priori and a posteriori tolerances depends on the context of discourse.

With the symbolic differentiation of a priori and a posteriori tolerances, different strategies of tolerance allocation could be applied in different scenarios. For example, in Figure 1-a, given two uncontrollable dimensions a and b, the controllable dimension $c = a + b = [2,5] + [1,3] = [3,8]$. In Figure 1-b, one extra controllable dimension b allows a tighter tolerance of c. $c = a + b = [2,5] + [3,1] = [5,6]$. The tolerance range of c is reduced from 5 to 1, which is smaller than the tolerance range of a. This indicates that the principle of selective assembly may be applied to achieve assembly. Selective assembly is a widely used process of sorting and selecting mating components in pairs so that high-precision assemblies can be achieved even with low-precision components. This method is valuable when individual components cannot be produced with tolerances small enough to be fully interchangeable in assembly such as specialized roller bearings with micrometer level tolerances. However, selective assembly is a manual process, which means it may only be used in low-volume high-value products. In a cost-conscious mass production environment, choosing flexible materials is the alternative, as discussed in Section 3.2.

3.2 Semantics of materials

In material property domain, the variation ranges for rigid materials are corresponding to proper intervals, and the ranges for flexible materials are to improper intervals. In the one-way clutch example of Figure 2, the distance vector b, the length of the spring s, and the radius of the ball r have the relation $r + s = b$. If variation ranges $[5.2,5.7]$ and $[7.8,8.0]$ are given to r and b respectively, the range for spring length s can be $[2.1,2.8]$, as in relation

$$\mathbf{r} + \mathbf{s} = [5.2,5.7] + [2.8,2.1] = [8.0,7.8] = \mathbf{b}\,. \tag{6}$$

It is interpreted as

$$(\forall r \in [5.2,5.7])(\forall b \in [7.8,8.0])(\exists s \in [2.1,2.8])(r + s = b)\,. \tag{7}$$

The spring provides a "cushion" to absorb variance. If a larger range $[7.8,8.5]$ is allowed for b, no flexible material is absolutely required for s to absorb variance, since the relation

$$\mathbf{r} + \mathbf{s} = [5.2,5.7] + [2.6,2.8] = [7.8,8.5] = \mathbf{b} \tag{8}$$

is interpreted as

$$(\forall r \in [5.2,5.7])(\forall s \in [2.6,2.8])(\exists b \in [7.8,8.5])(r + s = b)\,. \tag{9}$$

As illustrated in Figure 3-a, the semantic difference between rigid and flexible material is differentiated by interval modalities. If the width of the interval $\mathbf{x} = [\underline{x}, \overline{x}]$ is defined as $\mathrm{wid}\,\mathbf{x} := |\overline{x} - \underline{x}|$, the flexibility of materials is quantified by the width of improper intervals. The relative width of an improper interval indicates how flexible the material is. Compressibility may be indicated by the index $\mathrm{I}^{-}(\mathbf{x}) = (\underline{x} - \overline{x})/\underline{x}$ and stretchability by $\mathrm{I}^{+}(\mathbf{x}) = (\underline{x} - \overline{x})/\overline{x}$. For example, in Figure 3-b, material $\mathbf{x}_1$ is more flexible than material $\mathbf{x}_2$, and $\mathbf{x}_2$ is more flexible than $\mathbf{x}_3$. Selection of rigid or flexible materials thus can be integrated into algebraic relations.

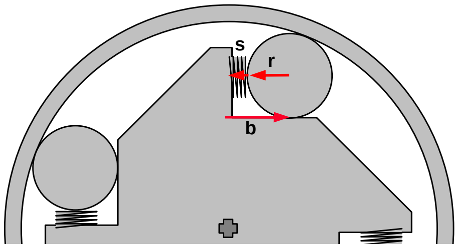

Figure 2: Variations form a closed loop in assembly

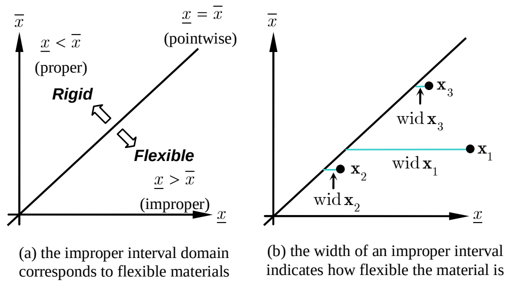

Figure 3: An inf-sup diagram is also a rigidity diagram

4. Semantic Tolerancing

In semantic tolerance models, algebraic relations among tolerances should be compatible with the semantics of engineering specifications. Based on the interpretability foundation of semantic tolerance modeling, a new dimensioning and tolerancing practice, semantic tolerancing, is proposed. The main difference between the semantic tolerancing and the commonly used tolerancing practice is that a priori and a posteriori tolerances are differentiated in the former one.

The major step of the semantic tolerancing practice is to differentiate a posteriori tolerance from a priori tolerance in symbols. Tolerances with universal modality are a priori tolerances, while those with existential modality are a posteriori tolerances. We use a minus-plus notation $x \mp \Delta$ in combination with the traditional plus-minus notation $x \pm \Delta$ for two modalities. If there is a closed tolerance chain $\sum_i d_i = z$ formed, dimensions on the left-hand side with the notation of $d_i \pm \Delta_i$ are a priori tolerances. Those with the notation of $d_i \mp \Delta_i$ are a posteriori. However, on the right-hand side of the chain, notations $z \pm \Delta$ and $z \mp \Delta$ are considered to be a posteriori and a priori tolerances respectively. If there is no closed tolerance chain in a drawing, $x \pm \Delta$ denotes a priori tolerance and $x \mp \Delta$ denotes a posteriori tolerance. For example, in Figure 4, the tolerance of a is a priori, and that of b is a posteriori. There is a closed tolerance chain $x + y = z$. Therefore, y and z are a priori, and x is a posteriori.

Assembly sequence can be inferred from the semantic tolerance chain stack-up. As illustrated in Figure 5-a, four dimensions a, b, g, and z are specified with a closed chain $a + g + b = z$. $a = 9 \pm 0.2$ and $b = 5 \mp 0.2$ imply that subassembly B is assembled after subassembly A. If the functional requirement of the working dimension g is not met, B needs to be adjusted. However, if the specifications are $a = 9 \mp 0.2$ and $b = 5 \pm 0.2$ as in Figure 5-b, A needs to be adjusted to meet the requirement of g. In Figure 5-c, $g = 3 \mp 0.2$ indicates that g is no longer functionally critical while a and b are.

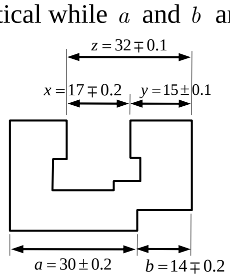

Figure 4: A priori and a posteriori tolerances in semantic tolerancing

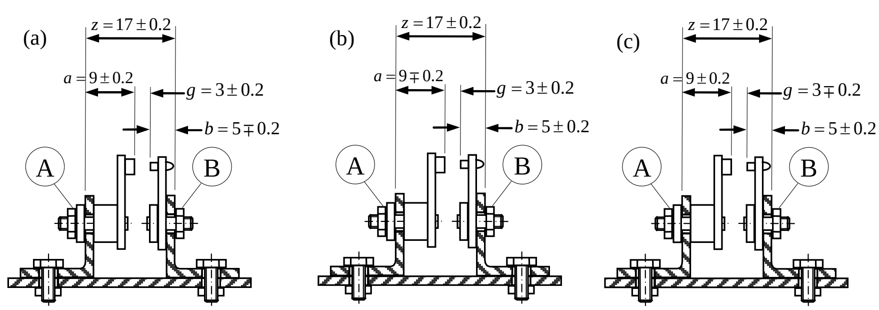

Figure 5: Semantic tolerancing implies assembly sequence

In semantic tolerancing, flexible and rigid material selection and assembly methods can be explicitly specified. Figure 6 illustrates flexible assembly and selective assembly examples of the Case III in Figure 1-c. The size tolerances of Part A and Part B are a posteriori. Both are larger than the size tolerance of Part C. Yet, three parts need to be assembled. The symbols of tolerance specifications in Figure 6-a indicate that flexible materials with the compressibility index at the level of $(0.3+0.3)/10.0=0.06$ need to be chosen for Parts A and B. If the variation ranges of a and b are reduced to ∓ 0.03 and selective assembly process is intended to be used, the a posteriori tolerance symbol captures the intent that A and B need to be sorted and paired, as in Figure 6-b.

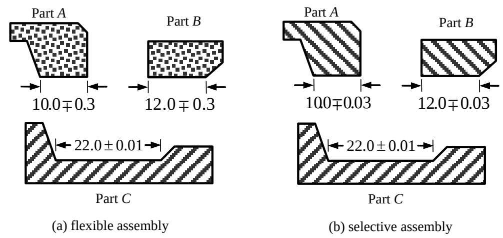

Figure 6: Semantic tolerancing captures intent of material selection and selective assembly

With the differentiation of existential and universal modalities associated with ranges, design intent can be captured in semantic tolerancing, such as how flexible materials are, whether a requirement is hard or soft, which sequence to take during assembly.

5. Concluding Remarks

In this paper, a dimensioning and tolerancing scheme, semantic tolerancing, with generalized or modal intervals is proposed. This new tolerancing practice allows for explicit differentiation between a priori and a posteriori tolerances. Based on interpretability principles, tolerancing semantics can be embedded in algebraic relations in order to support better design and manufacturing specifications. Symbolically, the new scheme captures more design intent such as physical property difference between rigid and flexible materials, rigidity of specifications, and sequence of assembly.

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