

Opening of Entrance Examination Problems Solutions Using a Graphic Calculator
(3) Examples of Synthesized Manipulation of Functions

Application: Trigonometric functions

Objective: Problems involving composite functions generally receive only formal treatment, and the meaning of problems is often not understood. Therefore, the graphic calculator can be used to understand the composition process.

Problem 1

$$I = 3\sin\theta \cos\theta - \sin\theta - \cos\theta, x = \sin\theta + \cos\theta$$

- a. Express I in terms of x .
- b-c. Determine the range of the values of x $b \leq x \leq c$
- d-e. Determine the maximum value (d) and the minimum value (e) of I .

Solution 1

(general solution)

Square both sides of $x = \sin\theta + \cos\theta$, and solve $x^2 = 1 + 2\sin\theta \cos\theta$ for $\sin\theta \cos\theta$.

$$\sin\theta \cos\theta = \frac{x^2 - 1}{2}$$

Substitute $\sin\theta + \cos\theta = x$, $\sin\theta \cos\theta = \frac{x^2 - 1}{2}$ in $I = 3\sin\theta \cos\theta - \sin\theta - \cos\theta$. Therefore,

$$I = \frac{3}{2}x^2 - x - \frac{3}{2} \dots\dots\dots (a)$$

Next, determine the range of the values of x from $x = \sin\theta + \cos\theta$.

Because $\sin\theta + \cos\theta = \sqrt{2} \sin\left(\theta + \frac{\pi}{4}\right)$, $-1 \leq \sin\left(\theta + \frac{\pi}{4}\right) \leq 1$, therefore $-\sqrt{2} \leq x \leq \sqrt{2}$

In short, $-\sqrt{2} \dots (b), \leq x \leq \sqrt{2} \dots (c) \dots\dots\dots (b \text{ and } c)$.

Next, determine the maximum value and the minimum value of I .

Rewrite (a) to:

$$I = \frac{3}{2}\left(x - \frac{1}{3}\right)^2 - \frac{5}{3} \quad \left(-\sqrt{2} \leq x \leq \sqrt{2}\right)$$

Here, $\frac{3}{2} > 0$, so:

the minimum value is $-\frac{5}{3}$ when $x = \frac{1}{3}$. $\dots\dots\dots (e)$

the maximum value is $\frac{3 + 2\sqrt{2}}{2}$ when $x = \sqrt{2}$. $\dots\dots\dots (d)$

Solution 2 (using a graphic calculator)

Like solution 1 determine $I = \frac{3}{2}x^2 - x - \frac{3}{2}$ (a)

Next, use the graphic calculator to determine the range of the values of x .

Enter $Y = \sin X + \cos X$ to the graphic calculator.

The initial settings of the View Window can be used.

Press **F6** (DRAW) to draw the graph.

Next, determine the maximum value **SHIFT F5** (G-SLV) **F2** (MAX) and minimum value **SHIFT F5** (G-SLV) **F3** (MIN).

Maximum value: 1.4142135623 Minimum value: -1.4142135623

Therefore, the range of the values of x is $-\sqrt{2} \dots (b), \leq x \leq \sqrt{2} \dots (c)$ (b and c)

Next, use the graphic calculator to determine the maximum value and minimum value of I .

Enter $Y = (3 \div 2)X^2 - X - (3 \div 2), [-\sqrt{2}, \sqrt{2}]$ to the graphic calculator.

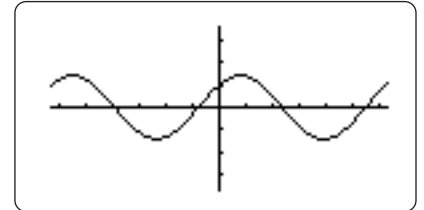
Initialize the View Window.

Press **F6** to draw the graph,

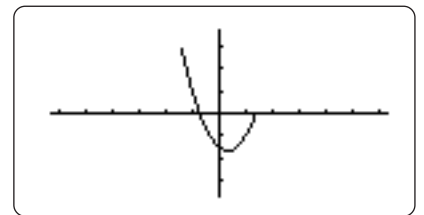
The graph shows the maximum value when $x = -\sqrt{2}$

Press **SHIFT F5** (G-SOLV) **F6** (\triangleright) **F1** (Y-CAL). $x =$ is displayed on the bottom left of the screen, so press $-\sqrt{2}$ **EXE** to obtain the value (2.9142135623). (d)

Next, press **SHIFT F5** (G-SLV) **F3** (MIN) to obtain the minimum value (-1.666...). (e)



01 Graph 1:
 $y = \sin x + \cos x (-6.3 \leq x \leq 6.3)$



02 Graph 2:
 $y = \frac{3}{2}x^2 - x - \frac{3}{2} (-\sqrt{2} \leq x \leq \sqrt{2})$

Solution 3 (using a graphic calculator)

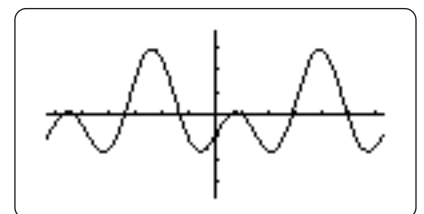
The procedure for determining (a) is similar to the method used in Solution 1, and the procedure for determining (b) and(c) is similar to the method used in Solution 2.

Enter $Y = 3 \sin X \cos X - \sin X - \cos X$ to the graphic calculator.

Initialize the View Window. Press **F6** to draw the graph.

Next, press **SHIFT F5** (G-SLV) **F3** (MIN) to obtain the minimum value (-1.666).... (e)

Press **SHIFT F5** (G-SLV) **F2** (MAX) to obtain the extreme value (0.08578643376). Then press the cursor right key (**→**) to obtain the maximum value (2.94142135623)....(d)



03 Graph 3:
 $y = 3 \sin x \cos x - \sin x - \cos x$