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Polarization changes in partially coherent electromagnetic beams propagating through turbulent atmosphere

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Abstract
In this paper, we study the effects of turbulent atmosphere on the degree of polarization of a partially coherent electromagnetic beam, which propagates through it. The beam is described by a 2×2 cross-spectral density matrix and is assumed to be generated by a planar, secondary, electromagnetic Gaussian Schell-model source. The analysis is based on a recently formulated unified theory of coherence and polarization and on the extended Huygens–Fresnel principle. We study the behaviour of the degree of polarization in the intermediate zone, i.e. in the region of space where coherence properties of the beam and the atmospheric turbulence are competing. We illustrate the analysis by numerical examples.

1. Introduction

It is generally believed that the changes in the degree of polarization of a random electromagnetic beam propagating through the turbulent atmosphere are negligible [1–3]. However, this opinion is based on the assumption that the beam is quasi-monochromatic. In recent years, it was found that when the beam is partially coherent the degree of polarization generally changes as the beam propagates even in free space, as James first showed in a seminal paper [4]. Somewhat similar results were later obtained by Gori *et al* in [5][2]. These investigations showed that such changes arise from correlation properties of the field in the source plane.

The effect of atmospheric turbulence on partially coherent beams has been studied up to now only within the framework of the scalar theory and, consequently, such treatments

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[2] The analysis of [5] was carried out in the space-time domain, using so-called 'beam coherence-polarization matrix'. Such a matrix is, however, restricted to description of quasi-monochromatic beams when only sufficiently small path differences are involved.

cannot provide any information about polarization properties of the beam. These early studies were mainly concerned with uses of partially coherent beams for free-space optical (FSO) communications [6, 7]. Later investigations have been carried out which elucidate some aspects of the propagation of random electromagnetic beams through the turbulent atmosphere [8, 9].

In a recent publication, the changes of the degree of polarization of the so-called Gaussian Schell-model beams in the far zone have been investigated [9]. The aim of this paper is to study such changes in the intermediate range, i.e. in the region of space where coherence properties of the beam and the atmospheric turbulence are competing. Our analysis is based on the unified theory of coherence and polarization [10] which makes it possible to determine how the degree of polarization of a light beam changes on propagation in any linear medium, deterministic or random [11]. The results are illustrated by a number of computed curves.

2. Propagation of the electric cross-spectral density matrix of an electromagnetic beam in the atmosphere

We begin by first considering the propagation of an electromagnetic wave in a linear medium.

It follows from Maxwell equations that the space-dependent part $\boldsymbol{E}(\boldsymbol{r};\omega)$ of a monochromatic electric field vector $\boldsymbol{E}(\boldsymbol{r};\omega)\exp(-\mathrm{i}\omega t)$ propagating in a linear medium satisfies the equation [12, section 13.1.1]

$$\nabla^2\boldsymbol{E}(\boldsymbol{r};\omega)+k^2n^2(\boldsymbol{r};\omega)\boldsymbol{E}(\boldsymbol{r};\omega)+\nabla[\boldsymbol{E}(\boldsymbol{r};\omega)\nabla\ln n^2(\boldsymbol{r};\omega)]=0, \tag{2.1}$$

where $k=2\pi/\lambda=\omega/c$, λ being the wavelength, ω the angular frequency, c the speed of light in vacuum, and $n(\boldsymbol{r};\omega)$ is the refractive index of the medium. We note that only the third term in equation (2.1) couples the Cartesian components of the electric field. Elementary arguments show that this coupling term may be neglected if the refractive index varies slowly with position; more precisely, if the fractional change $|\Delta n/n|$ is much smaller than unity in distances of the order of the wavelength. Under these circumstances, we may replace equation (2.1) by the equation

$$\nabla^2\boldsymbol{E}(\boldsymbol{r};\omega)+k^2n^2(\boldsymbol{r};\omega)\boldsymbol{E}(\boldsymbol{r};\omega)=0. \tag{2.2}$$

This equation shows that the three Cartesian components E_x, E_y, E_z of the electric field $\boldsymbol{E}$ then propagate independently of each other in the sense that each satisfies the equation

$$\nabla^2E_j(\boldsymbol{r};\omega)+k^2n^2(\boldsymbol{r};\omega)E_j(\boldsymbol{r};\omega)=0, \qquad (j=x,y\text{ or }z). \tag{2.3}$$

However, they may be coupled by boundary conditions.

Suppose that the field is beam-like and propagates from the plane $z=0$ into the half-space $z>0$ close to z-axis, where it encounters turbulent atmosphere. Let $\boldsymbol{r}=(\boldsymbol{\rho},z)$ be the position vector at a point in the half-space $z>0$, $\boldsymbol{\rho}$ denoting a two-dimensional transverse vector perpendicular to the direction of propagation of the beam (figure 1). If $\boldsymbol{E}^{(0)}(\boldsymbol{\rho}',0;\omega)$ represents the electric field vector at the point $(\boldsymbol{\rho}',0)$ in the plane $z=0$, which we will refer to as the source plane, the field at any point in the half-space $z>0$ into which the beam is assumed to propagate and which contains the turbulent atmosphere can be expressed by the following well-known expression based on the so-called extended Huygens–Fresnel principle [13, section 12.2]:

$$\boldsymbol{E}_j(\boldsymbol{\rho},z;\omega)=-\frac{\mathrm{i}k\exp(\mathrm{i}kz)}{2\pi z}\iint\boldsymbol{E}_j^{(0)}(\boldsymbol{\rho}';\omega)\exp\left[\mathrm{i}k\frac{(\boldsymbol{\rho}-\boldsymbol{\rho}')^2}{2z}\right]\times\exp(\psi(\boldsymbol{\rho},\boldsymbol{\rho}',z;\omega))\,\mathrm{d}^2\boldsymbol{\rho}', \qquad (j=x,y), \tag{2.4}$$

the integration extending over the source plane.

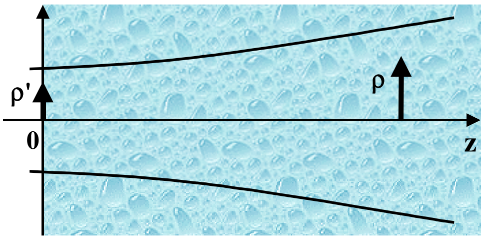

Figure 1. Illustrating the notation relating to propagation of a beam through a turbulent atmosphere.

In this formula ψ is a random phase factor which represents the effect of the turbulent atmosphere on a monochromatic spherical wave.

Suppose now that the beam is not monochromatic but is polychromatic and partially coherent. It must then be described by a correlation matrix rather than by the field vector. Such a matrix is defined as [10]

$$\overleftrightarrow{W}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z; \omega) \equiv W_{ij}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z; \omega) = \langle E_i^*(\boldsymbol{\rho}_1, z; \omega) E_j(\boldsymbol{\rho}_2, z; \omega)\rangle, \quad (i = x, y;\ j = x, y) \tag{2.5}$$

where the asterisk denotes the complex conjugate and the angular brackets represent the average over an ensemble of realizations of the electric field [14, section 4.7].

The elements of the cross-spectral density matrix at two points $(\boldsymbol{\rho}_1, z)$ and $(\boldsymbol{\rho}_2, z)$ in a transverse plane $z = \text{const} > 0$ may be obtained on substituting from equation (2.4) into equation (2.5) and one then finds that

$$W_{ij}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z; \omega) = \left(\frac{k}{2\pi z}\right)^2 \iint \mathrm{d}^2\rho_1' \iint \mathrm{d}^2\rho_2'\, W_{ij}^{(0)}(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2'; \omega) \times \exp\left[-\mathrm{i}k\frac{(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_1')^2 - (\boldsymbol{\rho}_2 - \boldsymbol{\rho}_2')^2}{2z}\right] \times \langle \exp[\psi^*(\boldsymbol{\rho}_1, \boldsymbol{\rho}_1', z, \omega) + \psi^*(\boldsymbol{\rho}_2, \boldsymbol{\rho}_2', z, \omega)]\rangle_m, \tag{2.6}$$

where $\langle \ldots \rangle_m$ denotes averaging over the ensemble of statistical realizations of the turbulent medium. It is assumed here that the fluctuations of light beam and of the turbulent atmosphere are mutually independent.

The elements of the electric cross-spectral density matrix $W_{ij}(\boldsymbol{\rho}, \boldsymbol{\rho}, z; \omega)$ i.e. with $\boldsymbol{\rho}_1 = \boldsymbol{\rho}_2 \equiv \boldsymbol{\rho}$ are evidently given by the expression

$$W_{ij}(\boldsymbol{\rho}, \boldsymbol{\rho}, z; \omega) = \left(\frac{k}{2\pi z}\right)^2 \iint d^2\rho_1' \iint \mathrm{d}^2\rho_2'\, W_{ij}^{(0)}(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2'; \omega) \times \exp\left[-\mathrm{i}k\frac{(\boldsymbol{\rho} - \boldsymbol{\rho}_1')^2 - (\boldsymbol{\rho} - \boldsymbol{\rho}_2')^2}{2z}\right] \times \langle \exp[\psi^*(\boldsymbol{\rho}, \boldsymbol{\rho}_1', z; \omega) + \psi(\boldsymbol{\rho}, \boldsymbol{\rho}_2', z; \omega)]\rangle_m, \tag{2.7}$$

where $W_{ij}^{(0)}(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2'; \omega) \equiv W_{ij}(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2', 0; \omega)$ is the electric cross-spectral density matrix in the source plane $z = 0$. An expression for the components of the matrix can be derived only by approximating the phase structure function, given by the expression in the angular brackets in equation (2.7). We will use the quadratic approximation [15]

$$\langle \exp[\psi^*(\boldsymbol{\rho}, \boldsymbol{\rho}_1', z; \omega) + \psi(\boldsymbol{\rho}, \boldsymbol{\rho}_2', z; \omega)]\rangle \cong \exp\left[\frac{-(\boldsymbol{\rho}_1' - \boldsymbol{\rho}_2')^2}{\rho_0^2(z)}\right], \tag{2.8}$$

where

$$\rho_0(z) = \left(0.55 C_n^2 k^2 z\right)^{-3/5} \tag{2.9}$$

is the spatial coherence radius of a spherical wave propagating in turbulence, whose behaviour is described by the Kolmogorov model and C_n^2 is the refractive index structure parameter [13, section 3.2.3] which characterizes the local strength of atmospheric turbulence[3].

Let us assume that the beam is generated by a planar secondary electromagnetic Schell-model source located in the plane $z = 0$ which we will refer to as the source plane and that it propagates into half-space $z > 0$ containing a turbulent medium. The elements of the electric cross-spectral density matrix (2.5) can then be expressed in the form [9]

$$W_{ij}^{(0)}(\boldsymbol{\rho}'_1, \boldsymbol{\rho}'_2; \omega) = \sqrt{S_i^{(0)}(\boldsymbol{\rho}'_1; \omega)}\sqrt{S_j^{(0)}(\boldsymbol{\rho}'_2; \omega)}\,\eta_{ij}^{(0)}(\boldsymbol{\rho}'_2 - \boldsymbol{\rho}'_1; \omega), \qquad (i = x, y;\ j = x, y). \tag{2.10}$$

In this formula $S_i^{(0)}$ is the spectral density of the component E_i of electric field in the source plane and $\eta_{ij}^{(0)}$ denotes the degree of correlation between the components E_i and E_j in that plane. All these quantities can be determined experimentally [17]. Also it can be shown that $\eta_{ij}^{(0)}$ satisfy the inequality $\left|\eta_{ij}^{(0)}\right| \leqslant 1$ for all values of their arguments (see appendix of [9]). Let us assume that

$$S_i^{(0)}(\boldsymbol{\rho}', \omega) = A_i^2 \exp\left(-\rho'^2/2\sigma_i^2\right), \qquad (i = x, y), \tag{2.11}$$

$$\eta_{ij}^{(0)}(\boldsymbol{\rho}'_2 - \boldsymbol{\rho}'_1, \omega) = B_{ij} \exp\left[-\frac{(\boldsymbol{\rho}'_2 - \boldsymbol{\rho}'_1)^2}{2\delta_{ij}^2}\right], \qquad (i = x, y;\ j = x, y), \tag{2.12}$$

where the coefficients A_i and B_{ij} are independent of position but may depend on the frequency. The same is true for the variances σ_i^2 and δ_{ij}^2. Moreover, the coefficients B_{ij} satisfy the relations (see [9], equations (2.5*a*)–(2.5*b*))

$$B_{ij} \equiv 1 \quad \text{when} \quad i = j \qquad |B_{ij}| \leqslant 1 \quad \text{when} \quad i \neq j \tag{2.13}$$

and

$$B_{ji} = B_{ij}^*.$$

We will impose the restriction that

$$\sigma_x = \sigma_y \equiv \sigma, \tag{2.14}$$

the degree of polarization will then be the same at all source points (see [5]).

On substituting from equations (2.11) and (2.12) into equation (2.7) and using equation (2.14) one obtains the following expressions for the elements of the electric cross-spectral density matrix of the beam in a plane $z = \text{const} > 0$,

$$W_{ij}(\boldsymbol{\rho}, \boldsymbol{\rho}, z; \omega) = \frac{A_i A_j B_{ij}}{\Delta_{ij}^2(z)} \exp\left[-\frac{\rho^2}{2\sigma^2 \Delta_{ij}^2(z)}\right], \qquad (i = x, y;\ j = x, y), \tag{2.15}$$

where

$$\Delta_{ij}^2 = 1 + \alpha_{ij} z^2 + 0.98\left(C_n^2\right)^{6/5} k^{2/5} \sigma^{-2} z^{16/5}, \qquad (i = x, y;\ j = x, y), \tag{2.16}$$

[3] Another approximation for the complex phase structure function for propagation of spherical wave has been given in [16] and is based on the Rytov approximation, with certain restrictions on the source parameters. However, one can show that the type of approximation and the particular choice of the model of the atmospheric turbulence do not introduce appreciable difference for the value of the degree of polarization.

with

$$\alpha_{ij} = \frac{1}{(k\sigma)^2}\left(\frac{1}{4\sigma^2} + \frac{1}{\delta_{ij}^2}\right). \tag{2.17}$$

In formula (2.15) $\Delta_{ij}^2(z)$ is the beam-spread factor (also called the expansion coefficient of the beam) determined from the Kolmogorov spectrum model [18]. It indicates how the beam spreads in free space. The third term on the right-hand side in expression (2.16) represents the spread caused by atmospheric turbulence[4].

3. The spectral degree of polarization of an electromagnetic Gaussian Schell-model (EGSM) beam propagating in a turbulent atmosphere

The degree of polarization $\mathcal{P}(\boldsymbol{\rho}, z; \omega)$ of a random electromagnetic beam at a point $(\boldsymbol{\rho}, z)$ is given by the expression [11]

$$\mathcal{P}(\boldsymbol{\rho}, z; \omega) = \sqrt{1 - \frac{4\mathrm{Det}\,\overleftrightarrow{W}(\boldsymbol{\rho}, \boldsymbol{\rho}, z; \omega)}{[\mathrm{Tr}\,\overleftrightarrow{W}(\boldsymbol{\rho}, \boldsymbol{\rho}, z; \omega)]^2}}, \tag{3.1}$$

where Det and Tr denote the determinant and the trace of the matrix $W(\boldsymbol{\rho}, \boldsymbol{\rho}, z; \omega)$ respectively. It follows at once from equations (3.1) and equations (2.10)–(2.12) that the degree of polarization in the source plane $z = 0$ is given by the expression

$$\mathcal{P}(\boldsymbol{\rho}', z = 0; \omega) = \frac{\sqrt{\left(A_x^2 - A_y^2\right)^2 + 4A_x^2A_y^2|B_{xy}|^2}}{A_x^2 + A_y^2}. \tag{3.2}$$

As already mentioned, the assumption (2.14) ensures that polarization across the source is uniform. The general expression for the degree of polarization of the EGSM beam at any distance $z > 0$ from the source plane can be derived by substituting equation (2.15) into equation (3.1). One then finds that

$$\mathcal{P}(\boldsymbol{\rho}, z; \omega) = \frac{[F(\boldsymbol{\rho}, z; \omega)]^{1/2}}{G(\boldsymbol{\rho}, z; \omega)}, \tag{3.3}$$

where

$$F(\boldsymbol{\rho}, z; \omega) = \left(\frac{A_x^2}{\Delta_{xx}^2(z)}\exp\left[-\frac{\rho^2}{2\sigma\Delta_{xx}^2(z)}\right] - \frac{A_y^2}{\Delta_{yy}^2(z)}\exp\left[-\frac{\rho^2}{2\sigma\Delta_{yy}^2(z)}\right]\right)^2 - \frac{4A_x^2A_y^2|B_{xy}|^2}{\Delta_{xy}^4(z)}\exp\left[-\frac{\rho^2}{\sigma\Delta_{xy}^2(z)}\right], \tag{3.4}$$

$$G(\boldsymbol{\rho}, z; \omega) = \frac{A_x^2}{\Delta_{xx}^2(z)}\exp\left[-\frac{\rho^2}{2\sigma\Delta_{xx}^2(z)}\right] + \frac{A_y^2}{\Delta_{yy}^2(z)}\exp\left[-\frac{\rho^2}{2\sigma\Delta_{yy}^2(z)}\right]. \tag{3.5}$$

At points on the optical axis ($\boldsymbol{\rho} = 0$) expressions (3.4) and (3.5) reduce to

$$F(0, z; \omega) = \left(\frac{A_x^2}{\Delta_{xx}^2(z)} - \frac{A_y^2}{\Delta_{yy}^2(z)}\right)^2 + \frac{4A_x^2A_y^2|B_{xy}|^2}{\Delta_{xy}^4(z)}, \tag{3.6}$$

[4] A similar expression for the beam spread was derived in [16] using a different atmospheric model.

Table 1. The form of two electric cross-spectral density matrices of the source.

Case 1	Case 2
$\overleftrightarrow{W}^{(0)} = \begin{pmatrix} W_{xx}^{(0)} & 0 \\ 0 & W_{yy}^{(0)} \end{pmatrix}$	$\overleftrightarrow{W}^{(0)} = \begin{pmatrix} W_{xx}^{(0)} & W_{xy}^{(0)} \\ W_{yx}^{(0)} & W_{xx}^{(0)} \end{pmatrix}$

$$G(0, z; \omega) = \frac{A_x^2}{\Delta_{xx}^2(z)} + \frac{A_y^2}{\Delta_{yy}^2(z)}. \tag{3.7}$$

It was shown in [8] and [9] that for sufficiently long propagation distances in the atmosphere, the degree of polarization approaches the value which it has in the source plane, i.e.,

$$\mathcal{P}(\boldsymbol{\rho}, z; \omega) \to \mathcal{P}(\boldsymbol{\rho}, z = 0; \omega) \quad \text{as} \quad kz \to \infty. \tag{3.8}$$

However, this result does not apply for propagation in free space; the asymptotic value as $kz \to \infty$ of the degree of polarization depends on the variances σ_i^2 and δ_{ij}^2 as well [9].

In the intermediate range, i.e. for $0 < z < \infty$, the behaviour of the degree of polarization of the beam propagating through the turbulent atmosphere is in general rather complicated. However, two special cases (listed in table 1) have already received attention in connection with free-space propagation. We will now revisit these two cases.

Case 1. $B_{xy} = B_{yx} = 0$. With this choice of beam parameters the electric cross-spectral density matrix at the source plane has only diagonal elements, i.e.,

$$W_{ij}^{(0)}(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2', z = 0; \omega) \equiv 0, \qquad i \neq j. \tag{3.9}$$

From equations (3.2) and (2.10) together with condition (2.14) it follows that the degree of polarization of the beam at points on the axis is now given by the expression (see also [8])

$$\mathcal{P}(0, z; \omega) = \frac{\left| \frac{A_x^2}{\Delta_{xx}^2} - \frac{A_y^2}{\Delta_{yy}^2} \right|}{\frac{A_y^2}{\Delta_{xx}^2} + \frac{A_x^2}{\Delta_{yy}^2}}. \tag{3.10}$$

It seems worth mentioning that in this case (no correlations between mutually orthogonal components of the electric field in the source plane) and with $A_x = A_y$ the source is unpolarized. It then follows from equation (3.10) that if also $\delta_{xx} = \delta_{yy}$, the beam remains unpolarized upon propagation. Further if $\delta_{xx} \neq \delta_{yy}$ the degree of polarization generally changes but has no zero values in the intermediate range.

Case 2. $B_{xy} \neq 0$, $B_{yx} \neq 0$, $\delta_{xx} = \delta_{yy}$, $\delta_{xy} \neq 0$. Next we consider another particular case of the electric cross-spectral density matrix (2.10). We assume that in the source plane the diagonal elements are equal to each other, i.e.,

$$W_{xx}^{(0)}(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2'; \omega) = W_{yy}^{(0)}(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2'; \omega). \tag{3.11}$$

We also assume that the *x* and *y* components of the electric field are correlated, i.e.,

$$W_{ij}^{(0)}(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2'; \omega) \neq 0, \qquad i \neq j. \tag{3.12}$$

From equations (2.10), (3.2), (3.11) and (3.12) it follows that the degree of polarization at point on the axis is, in this case, given by the expression

$$\mathcal{P}(0, z; \omega) = \frac{\sqrt{\left(A_x^2 - A_y^2\right)^2 + 4A_x^2 A_y^2 B_{xy}^2 \frac{\Delta_{xx}^4(z)}{\Delta_{xy}^4(z)}}}{A_x^2 + A_y^2}. \tag{3.13}$$

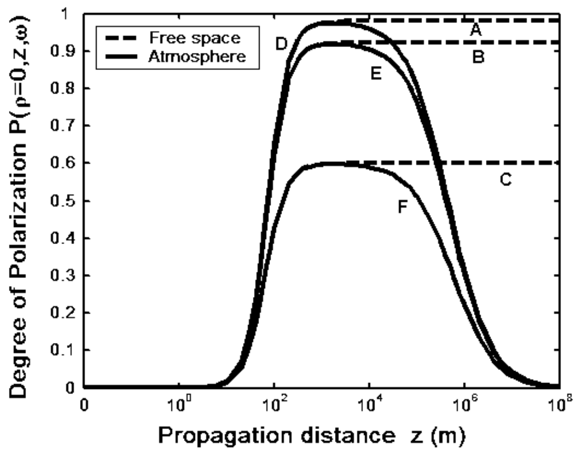

Figure 2. The change of the degree of polarization of an electromagnetic Gaussian Schell-model beam, calculated from equation (3.10). Curves A, B and C pertain to propagation in free space; D, E and F pertain to propagation in atmospheric turbulence. The source parameters were taken as $\omega = 3 \times 10^{15}$ rad m^{-1} ($\lambda = 0.628$ μm), $A_x^2 = A_y^2 = 1$, $B_{xy} = 0$, $\sigma_x = \sigma_y = \sigma = 5$ cm, $\delta_{xx} = 0.1$ mm. Curves A and D: $\delta_{yy} = 0.2$ mm; B and E: $\delta_{yy} = 0.5$ mm; C and F: $\delta_{yy} = 1$ mm. The refractive index structure parameter was chosen as $C_n^2 = 10^{-13} m^{2/3}$.

If equations (3.11) and (3.12) are satisfied and if $|B_{xy}| = 1$ then the source is fully polarized. According to appendix B of [5] the necessary condition $\delta_{xx} = \delta_{yy} = \delta_{xy}$ must then also be satisfied. This leads to the conclusion that the beam generated by such a source stays fully polarized on propagation at any distance z, both in free space and in atmospheric turbulence in agreement with usual belief.

4. Numerical examples

To illustrate the preceding analysis, we calculated the variation in the degree of polarization of electromagnetic Gaussian Schell-model beam along the beam axis for selected values of the source parameters. The results are shown in figures 2–6. Curves relating to propagation in free space are plotted for comparison.

Figures 2–4 show the changes in the degree of polarization for case 1 (see table 1). From these figures one can see the effect arising from the differences between the values of rms widths δ_{xx} and δ_{yy}. In particular, figure 2 shows the behaviour of the degree of polarization of the beam generated by a completely unpolarized source $\left(A_x^2 = A_y^2 = 1\right)$. It is to be noted that after sufficiently long propagation distance the beam becomes completely unpolarized again. Figures 3 and 4 show the corresponding results for beams generated by partially polarized sources.

Figures 5 and 6 show the changes in the degree of polarization of the beam, pertaining to case 2 (see table 1) and illustrate the effect introduced by the difference between rms widths δ_{xx} and δ_{xy}.

It is of interest to note that in case 2, as a consequence of the presence of correlations between the x and y components of the electric field, the degree of polarization does not become zero along the propagation path, in contrast with the situation noted in case 1 where it may vanish at some points as the beam propagates.

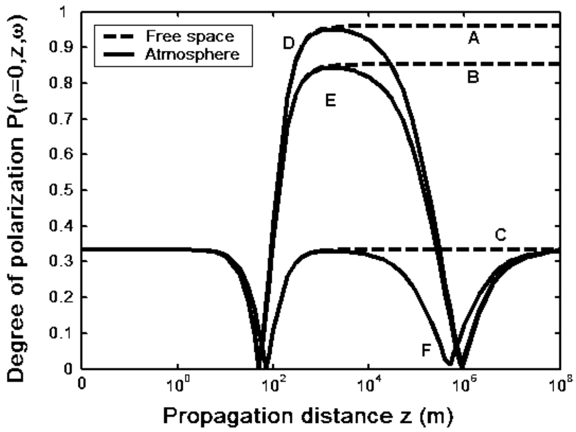

Figure 3. Same as for figure 2, but with $A_x^2 = 2$.

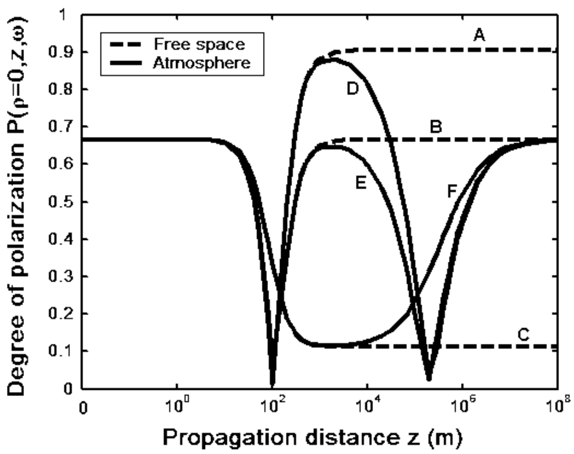

Figure 4. Same as figure 2, but with $A_x^2 = 5$.

5. Discussion

The two pronounced changes in the degree of polarization of an electromagnetic Gaussian Schell-model beam on the optical axis (illustrated in figures 2–6) may be explained in the following way. One can rewrite equations (3.3), (3.6) and (3.7) in the equivalent form

$$\mathcal{P}(0, z; \omega) = \frac{\sqrt{\left(\frac{A_x^2}{A_y^2}\frac{\Delta_{yy}^2(z)}{\Delta_{xx}^2(z)} - 1\right)^2 + 4|B_{xy}|^2 \frac{A_x^2}{A_y^2}\frac{\Delta_{yy}^4(z)}{\Delta_{xy}^4(z)}}}{\frac{A_x^2}{A_y^2}\frac{\Delta_{yy}^2(z)}{\Delta_{xx}^2(z)} + 1}, \tag{5.1}$$

which in the source plane $z = 0$ reduces to

$$\mathcal{P}(0, 0; \omega) = \frac{\sqrt{\left(\frac{A_x^2}{A_y^2} - 1\right)^2 + 4|B_{xy}|^2 \frac{A_x^2}{A_y^2}}}{\frac{A_x^2}{A_y^2} + 1}. \tag{5.2}$$

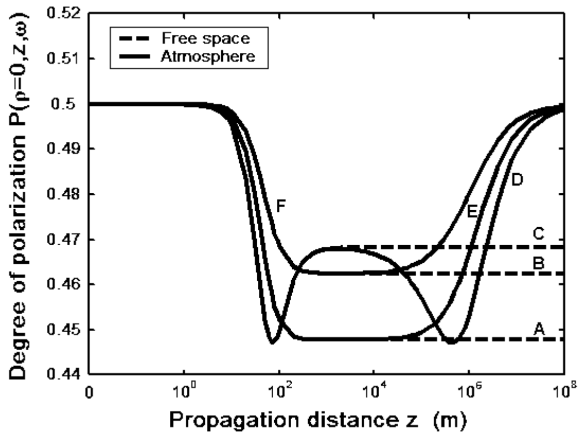

Figure 5. The change of the degree of polarization of an electromagnetic Gaussian Schell-model beam, calculated from equation (3.13). Curves A, B and C pertain to propagation in free space; D, E and F pertain to propagation in atmospheric turbulence. The source parameters were taken as $\omega = 3 \times 10^{15}$ rad m^{-1} ($\lambda = 0.628$ μm), $A_x^2 = A_x^2 = \frac{1}{2}$, $B_{xy} = \frac{1}{2}$, $\sigma_x = \sigma_y = \sigma = 5$ cm, $\delta_{xx} = 0.1$ mm. Curves A and D: $\delta_{xy} = 0.2$ mm, B and E: $\delta_{xy} = 0.5$ mm, C and F: $\delta_{xy} = 1$ mm. The refractive index structure parameter was chosen as $C_n^2 = 10^{-13} m^{2/3}$.

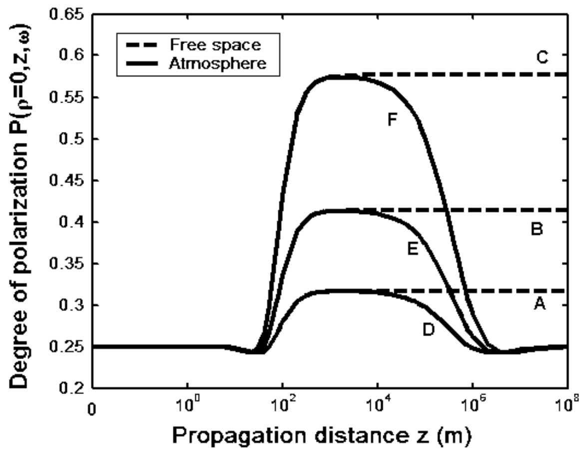

Figure 6. Same as figure 5, but with $B_{xy} = \frac{1}{2}$.

It is evident from equation (5.2) that the degree of polarization of an electromagnetic Gaussian Schell-model beam in the source plane is completely specified by the ratio $A_x^2 \big/ A_y^2$ and by B_{xy}. However, as the beam propagates, the degree of polarization also becomes a function of the two ratios $\Delta_{yy}^2(z) \big/ \Delta_{xx}^2(z)$ and $\Delta_{yy}^2(z) \big/ \Delta_{xy}^2(z)$, as one can see from equation (5.1).

On propagation in free space the spreads $\Delta_{xx}(z)$, $\Delta_{yy}(z)$ and $\Delta_{xy}(z)$ which depend on the non-equal correlation coefficients δ_{xx}, δ_{yy} and δ_{xy} respectively (see equations (2.16) and (2.17)), start to increase at different rates. Hence the two ratios mentioned above change with the distance z *independently* of each other, affecting the value of the degree of polarization. However, in the far zone all the beam spreads become quadratic in z, i.e. they approach the values $\alpha_{ij} z^2$ $(i, j = x, y)$ and the relative spreads acquire the constant values $\Delta_{yy}^2(z) \big/ \Delta_{xx}^2(z) = \alpha_{yy}/\alpha_{xx}$ and $\Delta_{yy}^2(z) \big/ \Delta_{xy}^2(z) = \alpha_{yy}/\alpha_{xy}$. Hence the degree of polarization

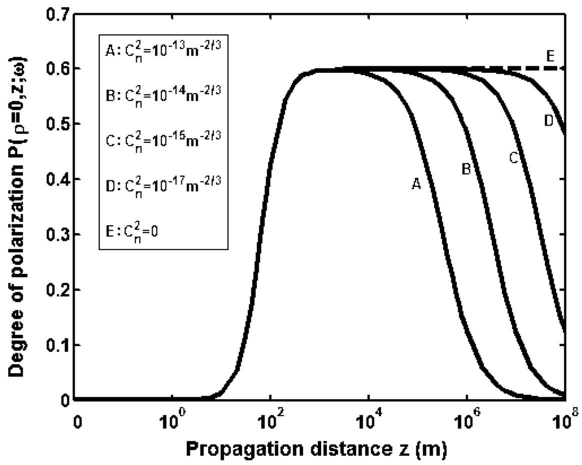

Figure 7. Illustrating the continuous transition of the degree of polarization of a completely unpolarized beam propagating in the atmospheric turbulence and in free space for several values of the refractive index structure parameter C_n^2. The source parameters are chosen the same as those for curves C and F in figure 2.

tends to a horizontal asymptote. This explains the only change of the degree of polarization in free-space propagation which is caused by the correlation properties of the source. It takes place at relatively small distances from the source (see the dashed curves in figures 2–6).

When the beam propagates in the atmospheric turbulence only over a small distance from the source, the strength of the turbulence is negligible and it cannot overcome the change in polarization caused by the correlation properties of the source. However, as the beam propagates sufficiently far, the effect of the atmosphere becomes dominant, leading to the following asymptotic expression for the beam spreads as $kz \to \infty$:

$$\Delta_{ij}^2 \approx 0.98\left(C_n^2\right)^{6/5} k^{2/5} \sigma^{-2} z^{16/5}, \qquad (i = x, y;\ j = x, y). \tag{5.3}$$

Using this formula it follows that $\Delta_{yy}^2(z) \big/ \Delta_{xx}^2(z) = \Delta_{yy}^2(z) \big/ \Delta_{xy}^2(z) = 1$. Hence the degree of polarization becomes independent of the distance z and remains a function of A_x^2/A_y^2 and B_{xy} only, just as it is in the source plane. We may conclude that the turbulent atmosphere, becoming stronger with increasing distance, 'washes out' all information about the correlation properties of the source. This explains why the degree of polarization acquires its original value (see the solid curves in figures 2–6), a fact which has already been addressed in [9].

We also note that there is a smooth continuous transition in the behaviour of the degree of polarization between propagation in the atmospheric turbulence and in free space as the local strength of turbulence (specified by the refractive index structure parameter C_n^2) decreases. In fact, if $C_n^2 \to 0$, the curves representing atmospheric propagation may be shown to tend locally (i.e. for any fixed distance z) to the corresponding free-space curves. This fact is illustrated in figure 7, where the degree of polarization is plotted (with a particular set of source parameters) for several values of C_n^2.

6. Concluding remarks

We have studied the changes in the degree of polarization of a random electromagnetic Gaussian Schell-model beam propagating in free space and through atmospheric turbulence.

Generally, the degree of polarization of a partially coherent beam propagating in a turbulent atmosphere is affected by two mechanisms. One is associated with the correlation properties of the source, which may be referred to as 'correlation-induced', and the other is due to the atmosphere, which is 'turbulence-induced' changes. The interplay between them is rather complicated as can be seen from the analysis presented in this paper. Since the combined effect of correlation-induced and turbulence-induced changes in the state of coherence and polarization of the beam can be controlled to some extent by the choice of the source parameters, the theory we described may find applications to problems involving inverse scattering in random media, including the turbulent atmosphere, and the development of more efficient schemes for imaging by laser radars and for free-space optical communication systems.

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