

The Effect of Partially Coherent Quasi-Monochromatic Gaussian-Beam on the Probability of Fade

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Abstract

The effect of a (spatially) partially coherent quasi-monochromatic lowest order Gaussian-beam on the direct detection system is studied in weak and strong atmospheric turbulence. The analytic expression for the scintillation index has been developed and analyzed as a function of the spatial correlation distance l_c and the correlation time τ_s , associated with the source, as well as the strength of turbulence and the integration time τ_d of the detector. The probability of fade is also discussed as a function of relative detector speed τ_d / τ_s , including limiting cases of slow ($\tau_d \gg \tau_s$) and fast ($\tau_d \ll \tau_s$) detector.

1. INTRODUCTION.

Free-space laser communication systems (FSO) are now attracting an increasing number of researchers and engineers¹⁻⁷. It was recently recognized that FSO systems can provide a cheaper solution (compared with fiber-optics option) for the so-called “last-mile” problem, i.e. the problem of data distribution among multiple local receivers after it was transferred from a distant source. Since such communication links are usually set up between buildings, the transmitted wave is being modulated by the turbulent medium (atmosphere and higher-temperature flows near walls⁴) which can result in deep fades of the signal and, consequently, high bit-error rates. Among all atmospheric effects the intensity fluctuations (scintillations) in the received signal are known to have the most dramatic effect on the performance of the FSO channel.

The classic approach for the mitigation of scintillations is based on the receiver angle diversity (realized through a large collecting aperture or multiple apertures)^{8,9}. However, it seems to be not convenient enough in this particular application, where the receiver size, its mobility and cost must be minimized. An attractive alternative is to exploit the transmitter angle diversity, or, in other words, to use a spatially coherent wave as a signal carrier. Although simply physically realizable

(usually with the help of a commercially sold diffuser placed over the laser exit) the new approach requires the development of a theoretical model for partially coherent beams propagating in atmospheric turbulence that could be useful for the FSO system design. Usually in literature existing on this account¹⁰⁻¹⁸ one can find the analytic results only for two limiting cases of fast and slow detection and only for one of the atmospheric conditions: weak fluctuations or saturation regime.

A model valid in all atmospheric fluctuations (weak-moderate-strong) was suggested by the authors for the particular case of a slow detector⁵⁻⁷ where it was demonstrated that the application of the diffuser can significantly improve the FSO system performance. The purpose of this paper is to extend previous results to the case of arbitrary detection speed.

2. BASIC MODEL.

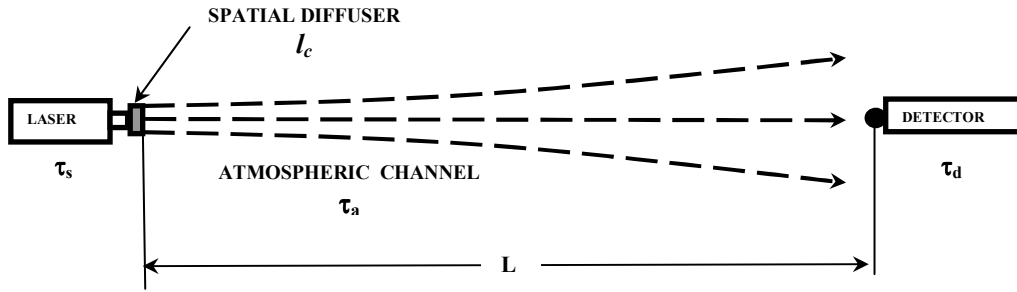


Fig. 1 Schematic diagram for a partially coherent Gaussian beam propagation.

A schematic diagram for the propagation of a partially coherent beam is shown in Fig. 1. We assume the transmitted beam wave is a TEM_{00} Gaussian-beam characterized by parameters⁸

$$\Theta_0 = 1 - \frac{L}{F_0}, \quad \Lambda_0 = \frac{2L}{kW_0^2}, \quad (2.1)$$

where $k = \frac{2\pi}{\lambda}$ (m^{-1}) is the laser wave number, $L(m)$ is the propagation distance to the collecting lens, $W_0(m)$ is the laser exit aperture radius. $F_0(m)$ is the phase front radius of curvature, which is infinite (collimated beam) for the case under study. We also assume that the source is quasi-monochromatic, with characteristic time (sec) $\tau_s \cong 1/B$, where $2B$ is its bandwidth.¹¹⁻¹³ For most laser sources¹² the coherence time is from the interval $10^{-12} \leq \tau_s \leq 10^{-3}$. The spatial diffuser placed over the laser exit is modeled as a thin complex phase screen with a Gaussian spectrum model⁵⁻⁷

$$\Phi_s(k) = \frac{\langle n_l^2 \rangle l_c^3}{8\pi\sqrt{\pi}} \exp\left(-\frac{1}{4} l_c^2 k^2\right), \quad (2.2)$$

where κ (m^{-1}) is the atmospheric wave number, l_c (m) is the lateral correlation radius of the Gaussian phase screen. In model (2.2), $\langle n_1^2 \rangle$ is the fluctuation in the index of refraction induced by the screen. We also introduce the non-dimensional quantity

$$q_c = \frac{L}{\kappa l_c^2}, \quad (2.3)$$

relating the strength of the diffuser (screen) to the first Fresnel zone. The diffuser generates a partially coherent Gaussian beam (of radius W_e and phase front radius of curvature F_e) that can be characterized by an effective set of beam parameters Λ_e, Θ_e calculated at a point detector:

$$\Lambda_e = \frac{\Lambda_1 N_s}{1 + 4q_c \Lambda_1}, \quad \Theta_e = \frac{\Theta_1}{1 + 4q_c \Lambda_1} \quad (2.4)$$

where Λ_1 and Θ_1 are the corresponding pupil plane parameters of a perfectly coherent beam given by

$$\Lambda_1 = \frac{\Lambda_0}{\Theta_0^2 + \Lambda_0^2}, \quad \Theta_1 = \frac{\Theta_0}{\Theta_0^2 + \Lambda_0^2} \quad (2.5)$$

and

$$N_s = 1 + \frac{4q_c}{\Lambda_0} \quad (2.6)$$

is the number of speckle cells in the transverse plane of the transmitted beam (the measure of the diffuser strength relative to the initial beam size).

The point photo-detector is located on the optical axis of the beam at distance L from the source. It is assumed the integration period (sec) is $\tau_d \cong 1/D$, where D is the data-rate. Integration period for modern photo-detectors is in the range $10^{-11} \leq \tau_s \leq 10^{-5}$.

3. SCINTILLATION INDEX.

We calculate the scintillation index $\sigma_I^2(L)$, caused first by the diffuser alone, and then caused by the combination of diffuser and atmospheric turbulence. In calculating the scintillation index, it is the integrated intensity that we must consider, taking into account the response time τ_d of the detector and the coherence time τ_s of the source. If the source coherence time is much smaller than the detector's integration time interval τ_d , i.e. $\tau_s \ll \tau_d$, temporal averaging of the fluctuating intensity occurs, which reduces the scintillation level through source aperture averaging.^{11-13, 15} In the case of a fast detector $\tau_s \gg \tau_d$, the detector is sensitive to intensity fluctuations of the source as well as those caused by the atmospheric turbulence. The characteristic time of atmospheric effects τ_a (sec) is generally somewhat greater than both scales τ_s and τ_d ($10^{-3} \leq \tau_a \leq 10^{-1}$);¹² therefore it will be ignored in the subsequent analysis.

Last, we point out that, owing to the focusing-like behavior of a strong diffuser, off-axis fluctuations cannot be accurately predicted by the Rytov theory except for the case of a weak diffuser. In the analysis below, we therefore only consider intensity fluctuations along the optical axis. However, in the case of a strong diffuser the fluctuations throughout most of the “beam” profile should be somewhat uniform.

3.1 Fast Detector Case: Free-Space Propagation.

Following Ref. 8 the on-axis scintillation index $\sigma_{I,\text{diff}}^2(L)$ due to the thin (with relative thickness Δz) complex-phase screen is defined by

$$\sigma_{I,\text{diff}}^2(L) = 8\pi^2 k^2 \Delta z \int_0^\infty \kappa \Phi_s(\kappa) e^{-\Lambda_1 L \kappa^2 / k} \left[1 - \cos\left(\frac{\Theta_1 L \kappa^2}{k}\right) \right] d\kappa, \quad (3.1)$$

By use of the Gaussian spectrum (2.2), this expression reduces to⁵

$$\sigma_{I,\text{diff}}^2(L) = \frac{\left(\frac{L \Theta_1}{k}\right)^2}{\frac{l_c^4}{16} (1 + 4 \Lambda_1 q_c)^2 + \left(\frac{L \Theta_1}{k}\right)^2}, \quad (3.2)$$

after the normalization

$$\frac{2\sqrt{\pi} \langle n_1^2 \rangle k^2 l_c \Delta z}{1 + 4 \Lambda_1 q_c} = 1. \quad (3.3)$$

It was previously shown⁴ that in the limiting cases (3.2) leads to $\sigma_{I,\text{diff}}^2(L) = 0$ for a weak diffuser ($l_c \rightarrow \infty$) and to $\sigma_{I,\text{diff}}^2(L) = 1$ for a strong diffuser ($l_c \rightarrow 0$) in agreement with that in Baykal and Plonus^{17,18}.

3.2 Integrated Intensity.

The integrated intensity can be represented by

$$E(\mathbf{r}, L) = \frac{1}{\tau_d} \int_{-\tau_d}^{\tau_d} I(\mathbf{r}, L; t) dt, \quad (3.4)$$

where $I(\mathbf{r}, L; t)$ is the instantaneous intensity. From linear system analysis, we recognize (3.4) as the output of an ideal integrator with impulse response function given by $h(t) = (1/\tau_d)U(\tau_d - |t|)$, where $U(t)$ is the unit step function. From linear systems theory, it is

known that the variance function of the output is related to the covariance function of the input by¹⁹

$$\sigma_E^2(L) = \sigma_{I,\text{diff}}^2(L) \left[\frac{1}{\tau_d} \int_{-\tau_d}^{\tau_d} \left(1 - \frac{|\tau|}{\tau_d} \right) |\gamma(\tau)|^2 d\tau \right], \quad (3.5)$$

where $|\gamma(\tau)|^2$ is the normalized temporal covariance function of the instantaneous source intensity. In arriving at (3.5), we have divided by the square of the mean intensity to obtain the scintillation index and we have assumed the covariance function of the input can be expressed as a product of the spatial covariance and temporal covariance. It is interesting that the above integral yields similar values for various models of $|\gamma(\tau)|^2$, e.g., (3.5) produces roughly the same results for either a Gaussian or Lorentzian spectrum¹¹. If we use a Lorentzian spectrum model, for example, it follows that $\gamma(\tau) = e^{-|\tau|/\tau_d}$. The result of doing so leads to the analytic expression

$$\sigma_E^2(L) = \sigma_{I,\text{diff}}^2(L) \left[\frac{\tau_s}{\tau_d} + \frac{1}{2} \left(\frac{\tau_s}{\tau_d} \right)^2 \left(e^{-2\tau_d/\tau_s} - 1 \right) \right]. \quad (3.6)$$

Equation (3.6) gives us the scintillation index of the integrated intensity as a function of coherence time of the source and response time of the detector. The two limiting cases of a slow and fast detector are readily deduced from (3.6), which yields

$$\sigma_E^2(L) = \begin{cases} \sigma_{I,\text{diff}}^2(L), & \tau_d \ll \tau_s \\ \frac{\tau_s}{\tau_d} \sigma_{I,\text{diff}}^2(L), & \tau_d \gg \tau_s \end{cases}. \quad (3.7)$$

In the lower expression (slow detector case), the ratio τ_s/τ_d approaches zero which eliminates all intensity fluctuations associated with the diffuser alone.

By following the technique used in Refs. 8, 9 and 20 for taking into account surface roughness irregularities, we express the instantaneous (normalized) intensity as $I = EI_{\text{atm}}$, where E is the integrated intensity of the source and I_{atm} is the random intensity due to the atmosphere and diffuser. Hence, we find $\langle I \rangle = \langle E \rangle \langle I_{\text{atm}} \rangle = 1$, $\langle I^2 \rangle = \langle E^2 \rangle \langle I_{\text{atm}}^2 \rangle = (1 + \sigma_E^2)(1 + \sigma_{I,\text{atm}}^2)$, and the scintillation index takes the form

$$\sigma_I^2(L) = \frac{\langle I^2 \rangle}{\langle I \rangle^2} - 1 = \sigma_{I,\text{atm}}^2 + \sigma_E^2 \left(1 + \sigma_{I,\text{atm}}^2 \right) \quad (3.8)$$

where σ_E^2 is defined by (3.6) and $\sigma_{I,\text{atm}}^2$ is the (fast) scintillation index of the atmosphere and diffuser effect taken up in the next section. In the weak fluctuation regime in the pupil plane of the receiver, the on-axis scintillation index of a partially coherent Gaussian-beam under Kolmogorov spectrum model is⁵

$$\sigma_{I,\text{atm,weak}}^2(L,0) = 3.86\sigma_1^2 \operatorname{Re} \left\{ i^{5/6} {}_2F_1 \left(-\frac{5}{6}, \frac{11}{6}; \frac{17}{6}; 1 - \Theta_e + i\Lambda_e \right) - \frac{11}{16} \Lambda_e^{5/6} \right\}, \quad (3.9)$$

where σ_1^2 is the Rytov variance, i is the imaginary unit, ${}_2F_1$ is the Hypergeometric function²¹, Re stands for the real part of the argument.

For strong atmospheric fluctuations the analytic formula

$$\sigma_{I,\text{atm,strong}}^2(0,L) = \exp \left[\frac{0.49\sigma_{I,\text{atm,weak}}^2}{\left(1 + 0.56\sigma_{I,\text{atm,weak}}^{12/5}\right)^{7/6}} + \frac{0.51\sigma_{I,\text{atm,weak}}^2}{\left(1 + 0.69\sigma_{I,\text{atm,weak}}^{12/5}\right)^{5/6}} \right] - 1 \quad (3.10)$$

can be applied⁶.

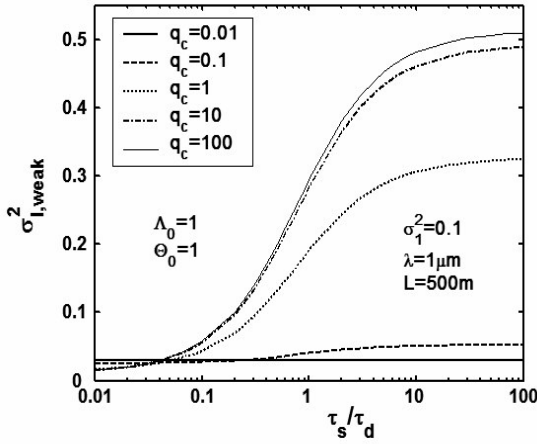


Fig. 2. Scintillation index vs. relative detector speed in weak turbulence.

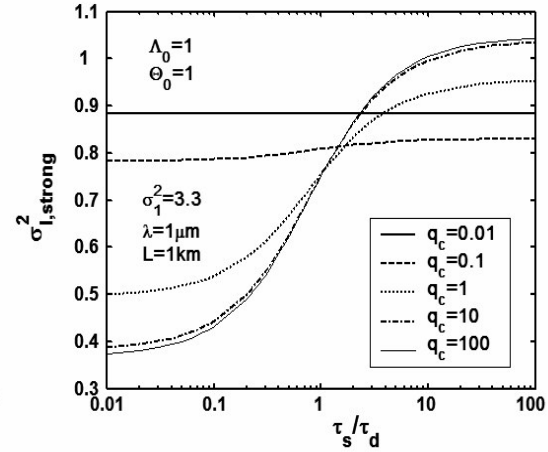


Fig. 3. Scintillation index vs. relative detector speed in focusing regime.

In Figures 2, 3 and 4 we display the scintillation index (3.8) in different atmospheric regimes (weak, moderate and strong) as a function of the detector integration time relative to the characteristic time of the diffuser, i.e. ratio τ_s/τ_d , called below “relative detector speed”. For all three plots the transmitted beam is collimated with normalized size $\Lambda_0=1$ and wavelength $1\mu\text{m}$. Five curves represent diffusers with different normalized correlation width q_c , in particular, the coherent beam is given by thick solid curves. In all three cases the effect of the diffuser on the system is positive only for small values of τ_s/τ_d (i.e. slow detector). However for each atmospheric regime and each diffuser there exist a maximum value of τ_s/τ_d (crossover with a thick solid curve) for which the diffuser can improve the system performance. Generally, systems with fast detection would be affected negatively in all fluctuation regimes.

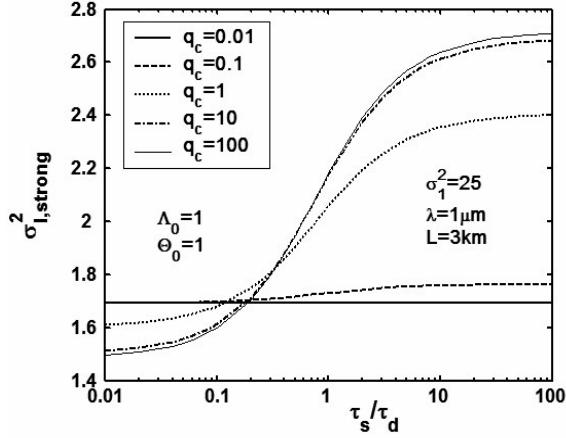


Fig. 4. Scintillation index vs. relative detector speed in strong turbulence.

A closer look at Fig. 3 (moderate atmospheric fluctuations) provides with an interesting fact that in this regime the diffuser can stay effective for quiet large values of τ_s/τ_d in striking difference with cases of weak and strong fluctuations. Therefore, for communication links 500m-1500m with the help of partial coherence it is possible to provide the data transfer at higher rates, for example, simply choosing $\tau_d \cong \tau_s$. We also note that the dashed curve ($q_c = 0.1$) stays below coherent beam (thick solid) curve independently of τ_s/τ_d .

4. PROBABILITY OF FADE.

In modeling of a spatially coherent beam propagating through atmospheric turbulence the resulted irradiance was represented⁹ by a product

$$I = XY, \quad (4.1)$$

accounting for a modulation process occurring between statistically independent intensities contributed by atmospheric irregularities of large-scale X (refracting effect) and small-scale Y (scattering effect).

When a spatially partially coherent beam is generated the model (4.1) must account for the third modulating component of the irradiance, that is, the irradiance due to diffuser surface irregularities E, introduced above by (3.4), i.e.

$$I = EXY. \quad (4.2)$$

The representation (4.2) was previously explored,^{8,9,20} however for a somewhat different problem (target characterization by laser radar). Again, intensities X, Y and E are statistically independent.

The measured field at the detector is the sum of many contributions of each type, therefore each intensity component (X, Y and ES) is governed by the Gamma PDF¹¹. The closed form of the PDF model for the total intensity I in (4.2) is unknown. However, it can be closely approximated by Gamma-Gamma distribution as shown below. In calculation of fade statistics we are interested only in the second moment of intensity I

$$\langle I^2 \rangle = \langle E^2 \rangle \langle X^2 \rangle \langle Y^2 \rangle, \quad (4.3)$$

where $\langle \cdot \rangle$ stands for the ensemble average. Under the assumption that the mean values of all

three processes are normalized by unity, i.e. $\langle E \rangle = \langle X \rangle = \langle Y \rangle = 1$, the scintillation index of I is given by

$$\sigma_I^2 = (\sigma_E^2 + 1)(\sigma_X^2 + 1)(\sigma_Y^2 + 1) - 1, \quad (4.4)$$

where σ_E^2 , σ_X^2 and σ_Y^2 are corresponding flux-variances of irradiance fluctuations. In particular, we have $\sigma_E^2 \rightarrow 0$ for a very weak and $\sigma_E^2 \rightarrow 1$ for a very strong diffuser.

Using the Gamma-Gamma approximation we write the scintillation index in the form

$$\sigma_I^2 \cong (\sigma_{X,ap}^2 + 1)(\sigma_{Y,ap}^2 + 1) - 1, \quad (4.5)$$

where flux-variances $\sigma_{X,ap}^2$ and $\sigma_{Y,ap}^2$ are to be determined. Setting $\sigma_{Y,ap}^2 \cong \sigma_Y^2$ we determine that $\sigma_{X,ap}^2 \cong \sigma_E^2 + \sigma_X^2 + \sigma_E^2 \sigma_X^2$. Introducing auxiliary parameters $\alpha = \frac{1}{\sigma_{X,ap}^2}$ and

$\beta = \frac{1}{\sigma_{Y,ap}^2}$ we can exploit the Gamma-Gamma PDF model⁹

$$P(I) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} I^{(\alpha+\beta)/2-1} K_{\alpha-\beta}(2\sqrt{\alpha\beta I}), \quad I > 0, \quad (4.6)$$

where $\Gamma(\cdot)$ is the Gamma-function, $K_\mu(\cdot)$ is the K-Bessel function of order μ . Then the probability of fade that the intensity is I_T units below the mean on-axis intensity is defined by

$$P(I < I_T) = \int_0^{I_T} P(I) dI \quad (4.7)$$

where the threshold I_T is set up a priori.

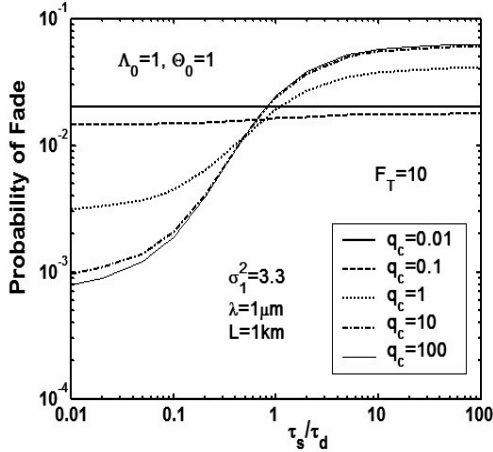


Fig. 5. Probability of fade vs. relative detector speed in focusing regime.

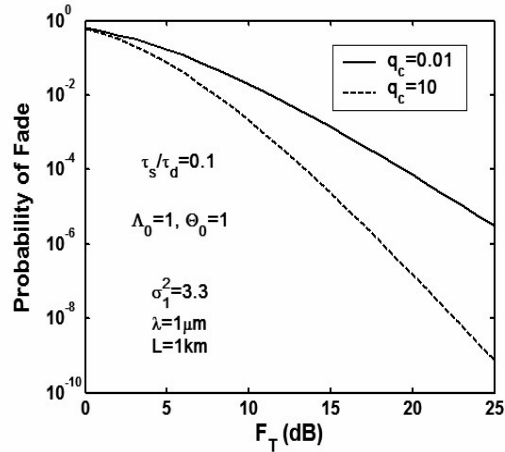


Fig. 6. Probability of fade vs. threshold level below the mean intensity.

In Fig. 5 we display the probability of fade for a FSO channel operating in a focusing regime vs. relative detector speed τ_s / τ_d . The point-detector is located on the optical axis. The threshold level is fixed at 10dB below the mean intensity $\langle I \rangle$ i.e. $F_T = 10 \log \left(\frac{I_T}{\langle I \rangle} \right) = -10\text{dB}$.

The direct consequence of the improvement in the scintillation index due to the diffuser (See Fig. 3) in this atmospheric regime results in the reduction of fade probability for relatively small time of detection. For example, choosing $\tau_s / \tau_d \approx 0.1$ we see that the probability of fade can be reduced by the application of the strong diffuser $q_c \gg 1$ up to an order of magnitude. Therefore, with a partially coherent beam it is possible to improve both data-rate and reliability level of the operating FSO system simultaneously.

In Fig. 6 the probability of fade is presented as a function of the threshold F_T for a perfectly coherent beam (solid curve) and for a partially coherent beam (dashed curve) at a fixed relative detector speed ($\tau_s / \tau_d = 0.1$). We see that for this fairly fast detector the probability of fade can be reduced up to several orders of magnitude with the help of a diffuser.

5. CONCLUDING REMARKS.

The proposed model for a scintillation index of a partially coherent Gaussian beam allows one to estimate fade statistics of the signal measured by a point detector of a variable response time. It might enable engineers to perform FSO channel optimization. In particular, it was demonstrated that the performance of a communication link operating in moderate atmospheric turbulence can be significantly improved with help of a partially coherent beam not only for the case of a slow detector,^{6,7} but also, for example $\tau_d \approx 10\tau_s$. However, in the case of faster detectors, i.e., $\tau_d < \tau_s$ the diffuser can introduce negative effect (see Fig.5). Thus, the decision to introduce the diffuser into the communication system should be made after an appropriate choice of the relative detector speed.

We note that in this study we focused our attention on the pupil plane analysis only, without taking into account the possibility to mitigate atmospheric effects by receiver aperture averaging, which could lead to an additional improvement.

REFERENCES

1. X. Zhu, J. Kahn, "Free-Space Optical Communication through Atmospheric Turbulence Channels," IEEE Trans. on Communications, Vol. **50**(8), 2002.
2. R. Tyson, "Bit-error rate for free-space adaptive optics laser communications", J. Opt. Soc. Am. A, Vol. **19**(4), 2002.
3. J. C. Ricklin and F. M. Davidson "Atmospheric optical communication with a Gaussian Schell beam", J. Opt. Soc. Am. A, Vol. **20**(5), 856-862, 2003.
4. S. Arnon, "Effects of atmospheric turbulence and building sway on optical wireless-communication systems", Opt. Lett. Vol. **28** (2), 129-131, 2003.
5. O. Korotkova, L. C. Andrews, R. L. Phillips "Speckle propagation through atmosphere: effects of a random phase screen at the source", Proc. SPIE **4821**, 2002.

6. O. Korotkova, L. C. Andrews, R. L. Phillips "A Model for a Partially Coherent Gaussian Beam in Atmospheric Turbulence with Application in Lasercom", (submitted to Opt. Eng.).
7. O. Korotkova, L. C. Andrews, R. L. Phillips "Phase Diffuser at the Transmitter for Lasercom Link: Effect of Partially Coherent Beam on the Bit-Error Rate." Proc. SPIE **4976**, 2003.
8. L. C. Andrews and R. L. Phillips, *Laser Beam Propagation Through Random Media* (SPIE Press, Bellingham, 1998).
9. L. C. Andrews, R. L. Phillips, and C. Y. Hopen, *Laser Beam Scintillation with Applications* (SPIE Press, Bellingham, 2001).
10. J. W. Goodman, "Statistical properties of laser speckle patterns," in *Laser Speckle and Related Phenomena*, J. C. Dainty, ed (Springer-Verlag, Berlin, 1975).
11. J. W. Goodman, *Statistical Optics* (Wiley, New York, 1985).
12. V. E. Zuev, V. A. Banakh, V. V. Pokasov, *Optics of the Turbulent Atmosphere* (Hidrometizdat, Leningrad, 1988).
13. L. Mandel, E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, Cambridge, 1995)
14. S. C. H. Wang and M. A. Plonus, "Optical beam propagation for a partially coherent source in the turbulent atmosphere," J. Opt. Soc. Am. **69**, 1297-1304, 1979.
15. L. Fante "Intensity fluctuations of an optical wave in a turbulent medium, effect of source coherence", Opt. Acta **28**, 1203-1207, 1981.
16. V. A. Banakh, V. M. Buldakov, and V. L. Mironov, "Intensity fluctuations of a partially coherent light beam in a turbulent atmosphere," Opt. Spektrosk. **54**, 1054-1059, 1983.
17. Y. Baykal, M. A. Plonus, S. J. Wang "The scintillations for weak atmospheric turbulence using a partially coherent source", Radio Science **18**, 551-556, 1983.
18. Y. Baykal and M. A. Plonus, "Intensity fluctuations due to a spatially partially coherent source in atmospheric turbulence as predicted by Rytov's method," J. Opt. Soc. Am. A **2**, 2124-2132, 1985.
19. C. W. Helstrom, *Probability and Stochastic Processes for Engineers*, 2nd ed. (Maxwell Macmillan International, New York, 1991).
20. V. Banakh and V. L. Mironov, *LIDAR in Turbulent Atmosphere*, (Artech House, Boston and London, 1984).
21. L. C. Andrews, *Special Functions of Mathematics for Engineers* (SPIE Press, Bellingham, 1998).