Phase Diffuser at the Transmitter for Lasercom Link: Effect of Partially Coherent Beam on the Bit-Error Rate.

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ABSTRACT

By using a complex phase screen model for a diffuser located at the transmitter, analytic expressions are developed for the scintillation index of a lowest order Gaussian-beam wave in the pupil plane of the receiver in weak and strong atmospheric conditions. The effect of partial coherence on the scintillation index is analyzed as a function of the propagation distance and the correlation length of the diffuser. The reduction in the scintillation level (due to the transmitter aperture averaging effect) is shown under all atmospheric conditions. The signal to noise ratio and the bit error rates (for OOK signal modulation) are discussed.

I. INTRODUCTION

The interest in the (spatially) partially coherent beam as a tool improving the performance of the laser communication systems was indicated recently in a number of publications [1-4]. Although the theoretical foundations of partial coherence in free space were established in early 60s [5, 6] there is still a lack of knowledge about the behavior of partially coherent beams in the atmospheric turbulence. During the 70s and early 80s the expressions for the second and the fourth order statistics (scintillation index, in particular) of partially coherent Gaussian beam field in the atmosphere were established by several authors in weak and saturation regimes [7-11]. However, only recently, based on the theory of optical scintillation for the coherent beams [12], the authors suggested the model for partially coherent beam valid for all atmospheric conditions (including the focusing regime) [13,14]. Besides the fact that the model does not have restrictions on the atmospheric spectrum, strength of turbulence and the degree of coherence, it also proves to be simple enough (in noticeable difference with other models) to be used for the analysis of optical system performance. Since [11] was published, it was well understood that a partially coherent source produces "transmitter" aperture averaging which can be used as an additional effect to the (commonly used) receiver aperture averaging. Therefore, changing the degree of spatial coherence (that is usually done by placing different diffusers over the transmitter aperture) there is a way to control (to some extent) the size of the collecting aperture, which is critical for the cost and flexibility of the system.

However, only a few attempts were recently made to calculate the bit-error rates [1, 15] of a partially coherent Gaussian beam propagating through the atmosphere. These studies were based on the quadratic approximation for the atmospheric spectrum, which could lead to the overestimation of actual performance level. We believe that with our model [13,14] one can predict the bit-error rate (as well as the probability of fades) of certain communication link with better accuracy.

One of the major drawbacks of partially coherent beams is the additional broadening as compared with a perfectly coherent beam, resulting in power loss at the receiver. This leads to the optimization problem for the signal to noise ratio with constraint, defining the optimal regime (the relation between the degree of coherence and transmitted power) for the operating system.

2. MODEL

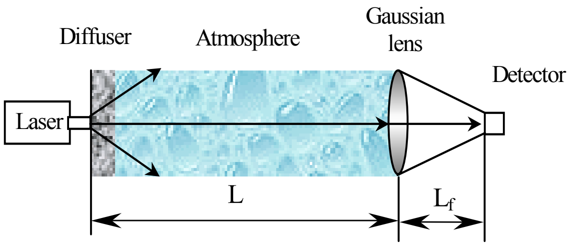

Figure 1. Propagation of partially coherent beam

A schematic diagram for the propagation of a partially coherent beam is shown in Figure 1 [13]. We assume the transmitted beam wave is a TEM_{00} Gaussian-beam wave characterized by parameters

$$\Theta_0 = 1 - \frac{L}{F_0}, \qquad \Lambda_0 = \frac{2L}{kW_0^2}, \tag{2.1}$$

where $k = \frac{2\pi}{\lambda}$ (m^{-1}) is the laser wave number (λ is chosen to be $1\mu m$ for all calculations), L(m) is the propagation distance to the collecting lens, F_0 (m) is the phase front radius of curvature, and W_0 (m) is the laser exit aperture radius. For illustrative purposes in all calculations for this paper we use the collimated beam ($F_0 = \infty$) with $W_0 = 0.025$m. The diffuser is modeled as a thin complex phase screen with a Gaussian spectrum model [16]

$$\Phi_s(\kappa) = \frac{\langle n_1^2 \rangle l_c^3}{8\pi\sqrt{\pi}} \exp\left(-\frac{1}{4} l_c^2 \kappa^2\right) \tag{2.2}$$

where κ (m^{-1}) is the atmospheric wave number, l_c (m) is the lateral correlation radius directly related to the variance σ_g^2 of the Gaussian phase screen also used in the literature [1], by

$$l_c^2 = 2\sigma_g^2, \tag{2.3}$$

In model (2.2), $\langle n_1^2 \rangle$ is the fluctuation in the index of refraction induced by the screen. We also introduce the non-dimensional quantity

$$q_c = \frac{L}{kl_c^2}, \tag{2.4}$$

relating the strength of the diffuser (screen) to first Fresnel zone. The effect of the diffuser is to create a "new" Gaussian beam that can be characterized by an effective set of beam parameters. Following [14], we introduce parameters Λ_e, Θ_e for the partially coherent beam (of radius W_e and phase front radius of curvature F_e) incident on the collecting lens:

$$\Lambda_e = \frac{\Lambda_1 N_s}{1 + 4q_c\Lambda_1}, \qquad \Theta_e = \frac{\Theta_1}{1 + 4q_c\Lambda_1} \tag{2.5}$$

where Λ_1 and Θ_1 are the corresponding pupil plane parameters of a perfectly coherent beam given by

$$\Lambda_1 = \frac{\Lambda_0}{\Theta_0^2 + \Lambda_0^2}, \qquad \Theta_1 = \frac{\Theta_0}{\Theta_0^2 + \Lambda_0^2} \tag{2.6}$$

and

$$N_s = 1 + \frac{4q_c}{\Lambda_0} \tag{2.7}$$

is the number of speckle cells in the transverse plane of the transmitted beam (the measure of the diffuser strength relative to the initial beam size).

The collecting Gaussian lens is described by the "soft aperture" radius W_G and focal length F_G. The nondimensional lens parameter $\Omega_G = \dfrac{2L}{kW_G^2}$ will be used as well.

3. SCINTILLATION INDEX. FLUX VARIANCE.

In the weak fluctuation regime in the pupil plane of the receiver, the on-axis scintillation index of partially coherent Gaussian-beam was calculated in [14]:

$$\sigma_B^2(L,0) = 3.86\sigma_1^2 \,\mathrm{Re}\left\{ i^{5/6} {}_2F_1\left(-\frac{5}{6},\frac{11}{6};\frac{17}{6};1-\Theta_e + i\Lambda_e\right) - \frac{11}{16}\Lambda_e^{5/6} \right\}, \tag{3.1}$$

where σ_1^2 is the Rytov variance, i is the imaginary unit, ${}_2F_1$ is the Hypergeometric function, Re stands for the real part of the argument.

Following [12] the flux variance of the irradiance fluctuations at the detector plane (valid for all atmospheric conditions) is

$$\sigma_I^2(L+L_f,\Omega_G) = \exp\left[\sigma_{\ln x}^2(L+L_f,\Omega_G) + \sigma_{\ln y}^2(L+L_f,\Omega_G)\right] - 1, \tag{3.2}$$

where $\sigma_{\ln x}^2$ is the flux variance associated with large-scale fluctuations given by

$$\sigma_{\ln x}^2(L+L_f,\Omega_G) = 0.49\sigma_1^2\left(\frac{\Omega_G-\Lambda_e}{\Omega_G+\Lambda_e}\right)^2 \cdot R \cdot \left(\frac{\eta_x}{1+\dfrac{0.4\eta_x(1+\Theta_e)}{\Lambda_e+\Omega_G}}\right)^{7/6}, \tag{3.3}$$

$$R = \frac{1}{3} - \frac{1}{2}(1-\Theta_e) + \frac{1}{5}(1-\Theta_e)^2 .$$

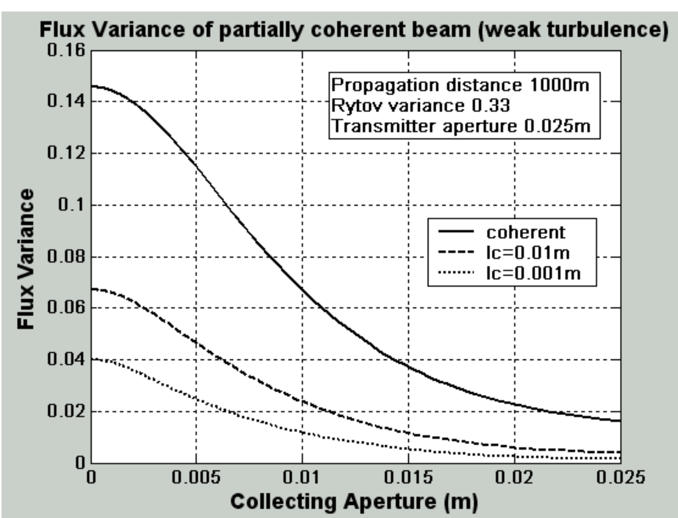

Fig. 2 Flux variance $\sigma_I^2(L+L_f,\Omega_G)$ as a function of the radius of the collecting lens W_G for coherent and partially coherent beams in weak turbulence.

The quantity η_x in (3.3) is the normalized large-scale cutoff frequency determined by the asymptotic behavior of $\sigma_{\ln x}^2$ in weak turbulence and saturation regime [12]:

$$\eta_x = \frac{R^{-6/7}(\sigma_B/\sigma_1)^{12/7}}{1+0.56\sigma_B^{12/5}} . \tag{3.4}$$

The small-scale flux variance $\sigma_{ln y}^2$ in (3.3) is defined by

$$\sigma_{\ln y}^2(L+L_f,\Omega_G) = \frac{1.27\sigma_1^2\eta_y^{-5/6}}{1+\dfrac{0.4\eta_y}{\Lambda_1+\Omega_G}}, \tag{3.5}$$

where the corresponding cutoff frequency is

$$\eta_y = 3(\sigma_1/\sigma_B)^{12/5}\left(1 + 0.69\sigma_B^{12/5}\right) \quad (3.6)$$

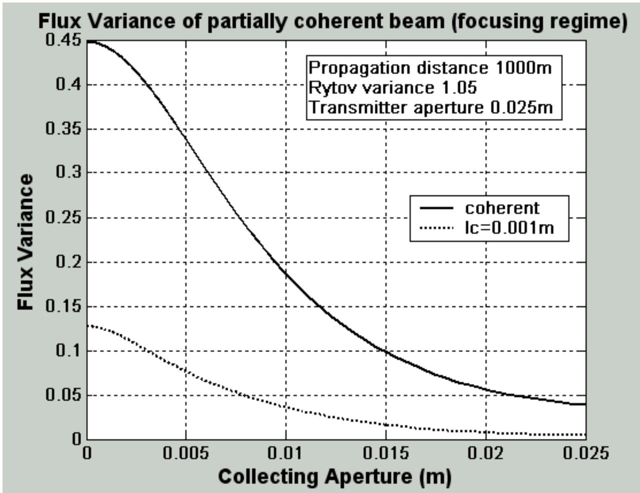

Fig. 3 Flux variance $\sigma_I^2(L+L_f,\Omega_G)$ as a function of the radius of the collecting lens W_G for coherent and partially coherent beams in focusing regime .

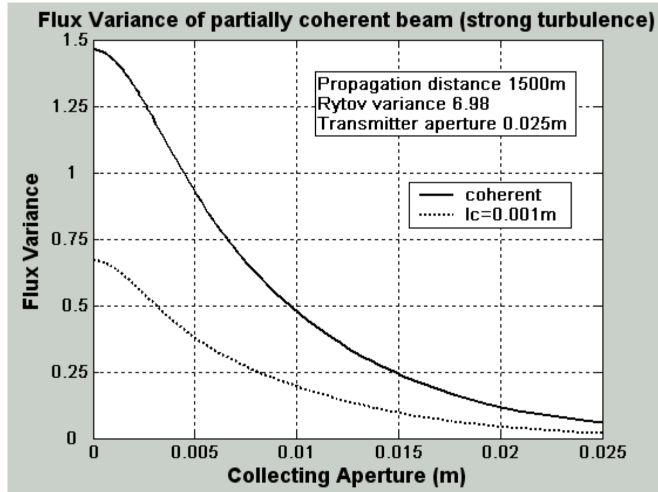

Fig. 4 Flux variance $\sigma_I^2(L+L_f,\Omega_G)$ as a function of the radius of the collecting lens W_G for coherent and partially coherent beams in strong turbulence.

Fig. 2 shows the flux variance (3.2) vs. W_G for a perfectly coherent beam (solid curve) and partially coherent beams: with $l_c = 0.01m$ (dashed curve) and with $l_c = 0.001m$ (dotted curve). The propagation distance L = 1000m, $C_n^2 = 10^{-14} m^{-2/3}$, leading to the Rytov variance $\sigma_1^2 = 0.033$. We see that in weak fluctuation regime there is a significant reduction of the flux, especially for a point aperture $W_G = 0$. With the increase of the collecting aperture size this effect decreases. In the analysis of the signal-to-noise ratio and the BER below we choose the collecting aperture radius $W_G = 0.01m$ to concentrate primarily on the averaging effect due to the transmitter. Similar curves are generated in moderate ($\sigma_1^2 = 1.05$) in Fig. 3 and in strong ($\sigma_1^2 = 6.98$) turbulence in Fig. 4, but as the strength of turbulence grows the absolute reduction of flux becomes less.

The aperture averaging factor of a partially coherent beam (AP) can be defined similarly to that of a perfectly coherent beam (AC), namely [16]

$$AP = \frac{\sigma_I^2\left(L+L_f,\Omega_G,q_c\right)}{\sigma_I^2\left(L+L_f,\infty,q_c\right)} \quad (3.7)$$

The dependence of AP on q_c is included in the formula above explicitly to show the contrast between AP and AC. There are two phenomena contributing to AP: aperture averaging by the collecting lens and “transmitter” aperture averaging due to partial coherence.

4. SIGNAL TO NOISE RATIO.

In free space the signal to noise ratio $SNRP_0$ of a partially coherent beam can be defined similarly to that of perfectly coherent beam [12]:

$$SNRP_0 = \frac{\langle i_s \rangle}{\sigma_N}, \quad (4.1)$$

where $\langle i_s \rangle$ is the mean value of the received signal current (proportional to the transmitted power) and σ_N^2 is the variance of the detector noise. Since partially coherent beams have greater divergence the received power depends upon the degree of coherence of the wave as well, i.e. more power is required for less coherent beams to sustain the same signal to noise ratio as the perfectly coherent beam (of the same size and phase front radius of curvature) produces. Therefore the relation between $SNRP_0$ and the corresponding free space signal to noise ratio of the coherent beam $SNRC_0$ is deduced from the beam size of partially coherent beam at the receiver, derived in [13]:

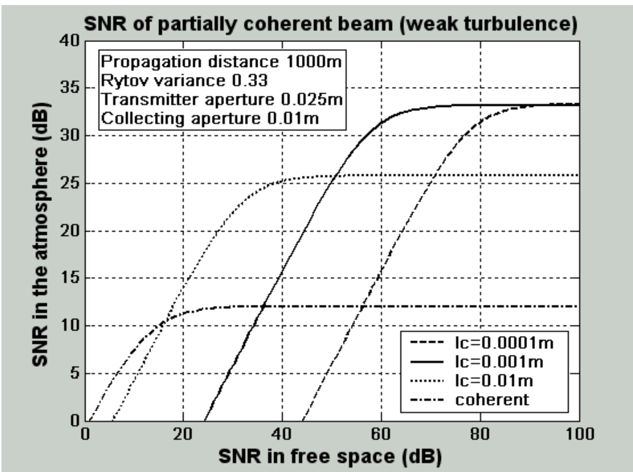

Fig. 5 <SNRP> (dB) as a function of $SNRC_0$ (dB) in weak turbulence for several values of l_c.

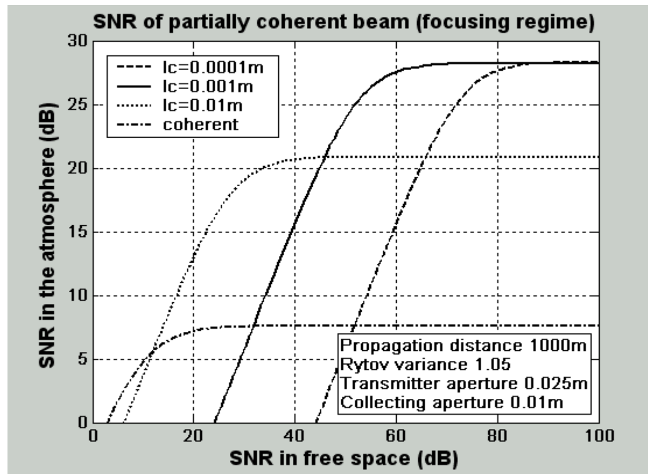

Fig. 6 <SNRP> (dB) as a function of $SNRC_0$ (dB) in focusing regime for several values of l_c.

$$SNRP_0 = \frac{SNRC_0}{\sqrt{PP_0/PC_0}} = \frac{SNRC_0}{\sqrt{1+4\,q_c\,\Lambda_1}}\,, \tag{4.2}$$

where PP_0 is the received power of the partially coherent beam and PC_0 is the power of coherent beam.

In the atmosphere the standard definition of SNR is adapted similarly:

$$\langle SNRP \rangle = \frac{SNRC_0}{\sqrt{\langle PP \rangle /PP_0 + \sigma_I^2 SNRC_0^2}} = \frac{SNRC_0}{\sqrt{1+4\,q_c\,\Lambda_1 + 1.63\,\sigma_I^{12/5}\,\Lambda_1 + \sigma_I^2 SNRC_0^2}} \tag{4.3}$$

where brackets are used for averaging.

In Fig. 5-7 we show the signal to noise ratio of the partially coherent beam <SNRP> (dB) as a function of $SNRC_0$ (dB) in different atmospheric conditions. We note that in all regimes the atmospheric SNR of a perfectly coherent beam (dashed-and-dotted curves everywhere) is always below 21-22dB, which corresponds to conventionally excepted BER of the operating system (on the order of 10^{-9}).

In weak fluctuation regime (Fig. 5) and in focusing regime (Fig. 6) the use of the partial coherence provides a noticeable improvement of the atmospheric SNR, namely in these regimes with $SNRC_0$ in the range 40-50 (dB) it is possible to obtain <SNRP> (dB) exceeding 21-22dB. We note that the saturation effect of <SNRP> is taking place for any strength of the diffuser and that of the turbulence (after some certain level of <SNRP> is attained the increase of

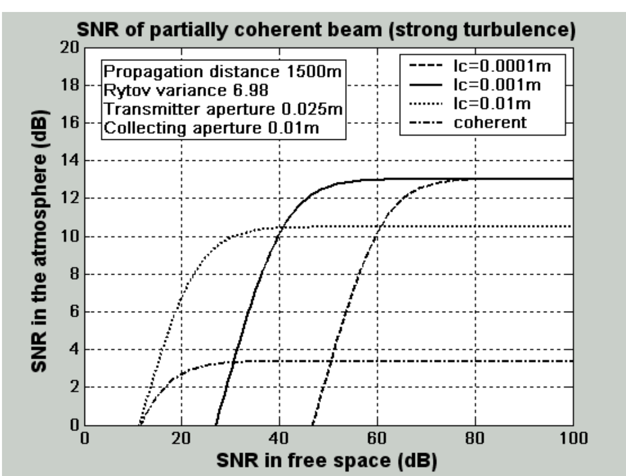

Fig. 7 <SNRP> (dB) as a function of $SNRC_0$(dB) in strong turbulence for several values of l_c.

power does not produce any additional effect). Therefore, for particular communication link and fixed atmospheric regime there exists the optimal diffuser. For example, in Fig. 5 for $SNRC_0$=30dB the best choice is the diffuser with $l_c = 0.01m$ (dotted curve) which corresponds to <SNRP> = 25(dB).

In Fig. 6 (focusing regime) the stronger diffuser together with greater amount of power are required to reach the BER=10^{-9}; here, for the fixed $SNRC_0$=30dB the diffuser with $l_c = 0.01m$ (dotted curve) would not provide with sufficient level of BER; for $SNRC_0$=50dB the diffuser with $l_c = 0.001m$ (solid curve) should be used.

Based on Fig. 7 (strong turbulence) we note that no matter how strong the diffuser, the required level of BER cannot be attained only with use of the partial coherence. In this regime only the combination of different tools (multi-aperture receivers, adaptive optics, partial coherence) would be efficient for the mitigation the atmospheric effects.

5. BIT-ERROR RATES.

Following [12] the probability of error (BER) for the Gaussian beam (OOK modulation scheme) in the atmospheric turbulence is based on the Gamma-Gamma probability density function of the intensity fluctuations:

$$BER = \frac{(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} \int_0^\infty erfc\left(\frac{\langle SNRP\rangle x}{2\sqrt{2}}\right) \cdot x^{\frac{\alpha+\beta}{2}-1} K_{\alpha-\beta}\left(2\sqrt{xa\beta}\right)dx \tag{5.1}$$

where <SNRP> is defined in (4.3), K(x) is K-Bessel function, erfc(x) is the complementary error function, α and β are two parameters of the distribution, related correspondingly to the large-scale x and the small-scale y intensity fluctuations:

$$\alpha = \frac{1}{\exp\left(\sigma_{\ln x}^2\right)-1}, \qquad \beta = \frac{1}{\exp\left(\sigma_{\ln y}^2\right)-1} \tag{5.2}$$

where $\sigma_{\ln x}^2$ and $\sigma_{\ln y}^2$ are given by (3.3) and (3.5).

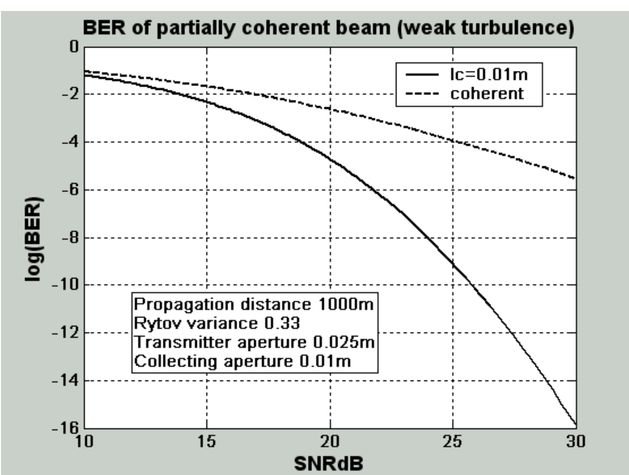

Fig. 8 The BER of partially coherent beams (logarithmic scale) as a function of <SNRP>(dB) in weak turbulence for coherent and partially coherent beams.

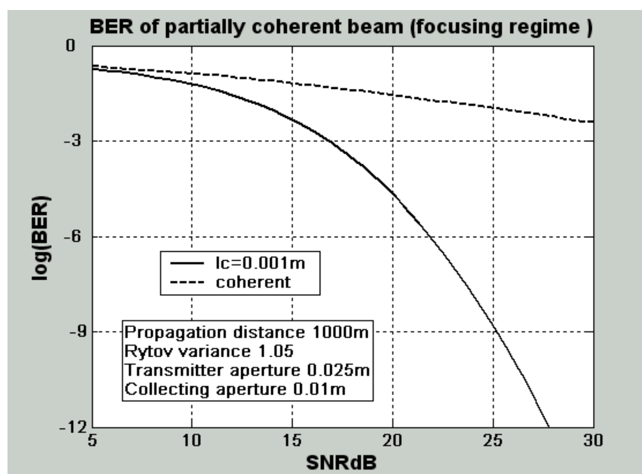

Fig. 9 The BER of partially coherent beams (logarithmic scale) as a function of <SNRP>(dB) in focusing regime for coherent and partially coherent beams

Although the probability (5.1) is valid in all atmospheric conditions we concentrate primarily on weak and moderate regimes where the partial coherence is proving to be very efficient. Using the conventional way to display the system performance, we plot BER (5.1) versus <SNRP>dB (see Fig. 8) for two beams: perfectly coherent (dashed curve) and partially coherent, with $l_c = 0.01m$ (solid curve) in weak turbulence ($\sigma_1^2 = 0.33$). The later diffuser is actually on the order of the optimal diffuser for this particular scenario (refer to Fig. 5).

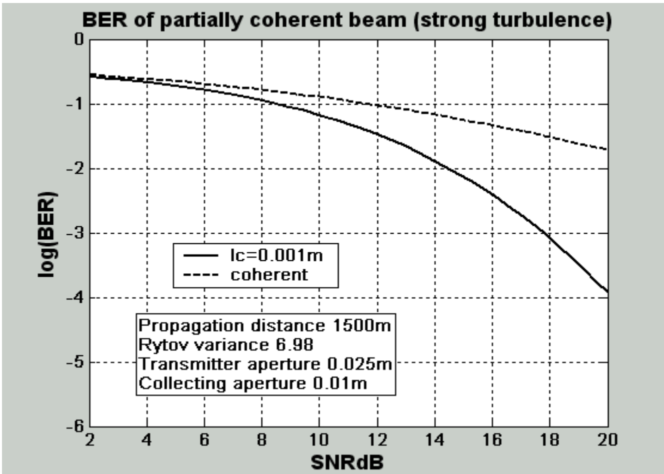

Fig. 10 The BER of partially coherent beams (logarithmic scale) as a function of <SNRP>(dB) in strong turbulence for coherent and partially coherent beams

If the transmitted power of the operating system is fixed (can not be adjusted) the partially coherent beam can improve the BER up to the several orders of magnitude over the distances under 1000m and values of C_n^2 on the order of $10^{-14}m^{-2/3}$, which corresponds to Rytov variance $\sigma_1^2 < 1$ (for example, in Fig. 8). This increase of BER is shown here to be critical (BER attains the value 10^{-9} at <SNRP>=25dB while perfectly coherent beam can not produce this BER with any transmitted power). See Fig. 5 to find the corresponding values of the free-space signal to noise ratio $SNRC_0$ for each beam.

On the other hand if a certain system performance is needed (BER should be set up permanently to some level) then the diffuser can be also used as a tool for the reduction of the transmitting power.

In Fig. 9 the BER of perfectly coherent beam (dashed curve) and partially coherent beam with $l_c = 0.001m$ (solid curve) are plotted in focusing regime ($\sigma_1^2 = 1.05$). This choice of the diffuser was discussed in Fig. 6. For <SNRP> = 25(dB) the BER = 10^{-9}, but it requires very high level of input power ($SNRC_0$ should be at least 50dB).

In Fig. 10 the BER of the same two beams as in Fig. 9 are displayed vs. <SNRP> in strong turbulence ($\sigma_1^2 = 6.98$). In this case partial coherence still reduces the BER but by the amount which is not sufficient for a good system performance.

With the decrease of correlation distance l_c parameter α given by (5.2) of the Gamma-Gamma distribution grows rapidly which might cause some difficulties in numerical integration of (5.1). In such cases single Gamma distribution might be used instead; hence the BER (5.1) can be replaced by

$$BER = \frac{1}{2}\frac{\gamma^\gamma}{\Gamma(\gamma)}\int_0^\infty erfc\left(\frac{\langle SNRP\rangle x}{2\sqrt{2}}\right)x^\gamma \exp(-\gamma x)dx \tag{5.3}$$

where γ is the parameter of the distribution, related to the flux-variance (3.2) by

$$\gamma = \frac{1}{\sigma_I^2\left(L + L_f, \Omega_G\right)} \tag{5.4}$$

The integral (5.3) predicts slightly higher levels of the BER compared with that of (5.1). The solid curve in Fig. 9 was generated with the help of this simplification.

6. CONCLUDING REMARKS

In this paper we made the attempt to calculate the main characteristics of performance of the communication system, (including the bit-error rates) using partially coherent beam in various atmospheric conditions. Generally partial coherence of the beam controls the part of the flux variance, related to large-scale fluctuations, namely, $\sigma^2_{\ln x}(L+L_f,\Omega_G)\to 0$ as $l_c \to 0$. In weak fluctuation regime the condition $\sigma^2_{\ln x}(L+L_f,\Omega_G)\cong 0.5\sigma^2_I(L+L_f,\Omega_G)$ holds, leading to the significant effect of partial coherence (strength of the diffuser). On the other hand, in strong regime we have $\sigma^2_{\ln x}(L+L_f,\Omega_G) << \sigma^2_I(L+L_f,\Omega_G)$, therefore the effect of the diffuser on the total flux variance is less. This leads to the corresponding effects on the signal to noise ratio and the BER.

As was noted in Section 3 of this paper, the aperture averaging factor AP of the beam depends on the degree of coherence of the transmitted beam as well as the collecting aperture size. Hence, partial coherence of the source (realizable by placing a relatively light and cheap diffuser over the laser exit aperture) can be used instead of more expensive and cumbersome collecting lens, especially in cases when the transmitter can be stationary but the receiver system has to be mobile.

7. REFERENCES

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