The Spectral Degree of Coherence of a Random Three-dimensional Electromagnetic Field

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Abstract. The complex spectral degree of coherence of a general random statistically stationary electromagnetic field is introduced in a manner similar to the way it is defined for beam-like fields: by means of Young's interference experiment. This quantity is measurable. The results are of particular interest for near-field optics.

OCIS codes: 030.1640 (Coherence), 030.6600 (Statistical Optics), 260.2110 (Electromagnetic Theory).

A quantitative measure of a degree of coherence of a directional (i.e. beam-like) field was introduced on the basis of a scalar theory by Zernike in a classic paper [1], which provided a basis for the development of modern coherence theory. Definition of the degree of coherence was later refined and generalized in several ways [2] but the essence of Zernike's approach was retained; namely, the degree of coherence of an optical field (more precisely its absolute value) at two points was taken to be proportional to the visibility of fringes in a Young's interference experiment, with pinholes at the two points.

In this talk we consider a general (i.e. not necessarily beam-like) randomly fluctuating electromagnetic field which propagates from a plane $z=0$ into the half-space $z>0$. We assume that the fluctuations are characterized by a statistical ensemble which is stationary, at least in the wide sense. The second-order statistical properties of the electric field may be characterized by the 3×3 cross-spectral density matrix (cf. [3], Sec. 6.6.1) of the electric field $\boldsymbol{E}=\left(E_x, E_y, E_z\right)$

$$\overleftrightarrow{W}(\boldsymbol{r}_1,\boldsymbol{r}_2,\omega)\equiv W_{\alpha\beta}(\boldsymbol{r}_1,\boldsymbol{r}_2,\omega)=\left\langle E_\alpha^*(\boldsymbol{r}_1,\omega)E_\beta(\boldsymbol{r}_2,\omega)\right\rangle, \qquad (\alpha=x,y,z;\ \beta=x,y,z), \tag{1}$$

where $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are the position vectors of two points in the half-space $z\geq 0$ and ω denotes the frequency. The subscripts α, β label Cartesian components with respect to two mutually orthogonal x, y – axes perpendicular to the z- direction and the angular brackets on the right of Eq. (1) denote the average, taken over a statistical ensemble of space-frequency realizations ([3], Sec. 4.7).

With the help of Young's interference experiment and the Luneberg diffraction formulas [4] we show that the (complex) spectral degree of coherence of the general three-dimensional electromagnetic field is given by the expression

$$\eta(\boldsymbol{r}_1,\boldsymbol{r}_2,\omega)\equiv\frac{Tr\,\overleftrightarrow{W}(\boldsymbol{r}_1,\boldsymbol{r}_2,\omega)}{\left[Tr\,\overleftrightarrow{W}(\boldsymbol{r}_1,\boldsymbol{r}_1,\omega)\,Tr\,\overleftrightarrow{W}(\boldsymbol{r}_2,\boldsymbol{r}_2,\omega)\right]^{1/2}}. \tag{2}$$

Both the modulus and the phase of η are measurable [5]. We illustrate the result by determining the spectral degree of coherence of blackbody radiation.

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