The Spectral Degree of Coherence of a Random Three-dimensional Electromagnetic Field

Olga Korotkova[1,2] and Emil Wolf[1,2,3]

[1]*Department of Physics and Astronomy, University of Rochester, Rochester, NY 14627*
[2]*School of Optics/CREOL, University of Central Florida, Orlando, FL 32816*
[3]*The Institute of Optics, University of Rochester, Rochester, NY 14627.*

Abstract

The complex spectral degree of coherence of a general random, statistically stationary, electromagnetic field is introduced in a manner similar to the way it is defined for a beam-like field, namely by means of Young's interference experiment. Both its modulus and its phase are measurable. We illustrate the definition by considering a blackbody radiation field. The results are of particular interest for near-field optics.

OSIC codes: *030.1640 (Coherence), 030.6600 (Statistical Optics), 260.2110 (Electromagnetic Theory), 260.3160 (Interference).*

1. Introduction.

A quantitative measure of a degree of coherence of a directional (beam-like) field was introduced on the basis of the scalar theory by Zernike in a classic paper [1] (see also [2]), which provided a basis for the development of modern coherence theory. Definition of the degree of coherence was later refined and generalized in several ways [3]–[7] but the essence of Zernike's approach was retained; namely, the degree of coherence of an optical field (more precisely its absolute value) at two points Q_1 and Q_2 was taken to be proportional to the visibility of fringes in a Young's interference experiment, with pinholes at the two points.

In the present paper we consider a general (i.e. not necessarily beam-like) statistically stationary electromagnetic field and show, with the help of the Rayleigh – Luneberg diffraction integrals [8], that for such fields it is also possible to take the visibility of fringes as a measure of coherence. The expression for the degree of coherence of a general three-dimensional statistically stationary electromagnetic field, which we now introduce, is a natural generalization of the definition introduced recently for beam-like electromagnetic fields [7].

We illustrate our main result by determining the degree of coherence of radiation emerging from an opening in a cavity containing a field in thermal equilibrium with the walls of the cavity, i.e. blackbody radiation. Our results may be expected to be useful for near-field optics because close to radiating sources and scatterers the optical fields are generally not beam-like.

2. The electric cross-spectral density of a random electromagnetic field propagating into a half-space.

Let us consider a randomly fluctuating electromagnetic field which propagates from a plane $z=0$ into the half-space $z>0$. We assume that the fluctuations are characterized by a statistical ensemble which is stationary, at least in the wide sense. The second-order statistical properties of the electric field may be characterized by the 3×3 cross-spectral density matrix (cf. [9], Sec. 6.6.1)

$$\overleftrightarrow{W}(\boldsymbol{r_1},\boldsymbol{r_2},\omega)\equiv W_{\alpha\beta}(\boldsymbol{r_1},\boldsymbol{r_2},\omega)=\left\langle E_{\alpha}^{*}(\boldsymbol{r_1},\omega)E_{\beta}(\boldsymbol{r}_2,\omega)\right\rangle, \qquad (\alpha=x,y,z;\ \beta=x,y,z), \tag{2.1}$$

where $\boldsymbol{r_1}$ and $\boldsymbol{r_2}$ are the position vectors of two points in the half-space $z\geq 0$ and ω denotes the frequency. The subscripts α,β label Cartesian components with respect to two mutually orthogonal x, y – axes perpendicular to the z- direction and the angular brackets on the right of Eq. (2.1) denote the average taken over a statistical ensemble of space-frequency realizations ([9], see Sec. 4.7).

Let us consider a typical member $\boldsymbol{E}(\boldsymbol{r},\omega)$ of the ensemble. It is known (see references in [10]) that the x, y – components on the boundary plane $z=0$, together with the outgoing condition at infinity in the half-space $z>0$, determine uniquely the electric field throughout the half – space $z>0$. The three Cartesian components of the electric field are given by the following formulas which appear to have been first derived by Luneberg [8]:

$$E_x(\boldsymbol{r},\omega) = -\frac{1}{2\pi}\int E_x(\boldsymbol{\rho},\omega)\,G_z(\boldsymbol{r},\boldsymbol{\rho})\,d^2\boldsymbol{\rho}, \tag{2.2a}$$

$$E_y(\boldsymbol{r},\omega) = -\frac{1}{2\pi}\int E_y(\boldsymbol{\rho},\omega)\,G_z(\boldsymbol{r},\boldsymbol{\rho})\,d^2\boldsymbol{\rho}, \tag{2.2b}$$

$$E_z(\boldsymbol{r},\omega) = \frac{1}{2\pi}\int \left[E_x(\boldsymbol{\rho},\omega)G_x(\boldsymbol{r},\boldsymbol{\rho}) + E_y(\boldsymbol{\rho},\omega)G_y(\boldsymbol{r},\boldsymbol{\rho})\right]d^2\boldsymbol{\rho}, \tag{2.2c}$$

where $\boldsymbol{\rho}$ is a two dimensional position vector of a point in the plane $z=0$ and G_x, G_y, G_z are the partial derivatives of the outgoing free-space Green's function

$$G(\boldsymbol{r},\boldsymbol{r}') = \frac{e^{ik|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|}, \tag{2.3}$$

i.e.

$$G_x(\boldsymbol{r},\boldsymbol{\rho}) = \frac{\partial}{\partial x'}\left[\frac{e^{ik|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|}\right]_{\boldsymbol{r}'=\rho} \tag{2.4}$$

with similar expressions for G_y and G_z,

$$k = \omega / c \tag{2.5}$$

denotes the wave number associated with the frequency ω, c being the speed of light in vacuum.

For later purposes we need an expression for the boundary value of the z-component of the electric field $E_z(\boldsymbol{\rho})$ in the plane $z=0$. It is given by the expression

$$\begin{aligned} E_z(\boldsymbol{\rho},\omega) &= \lim_{z\to 0} E_z(\boldsymbol{\rho},z,\omega) \\ &= \frac{1}{2\pi}\int \boldsymbol{E}(\boldsymbol{\rho}',\omega)\cdot(\boldsymbol{\rho}'-\boldsymbol{\rho})\frac{ik|\boldsymbol{\rho}'-\boldsymbol{\rho}|-1}{|\boldsymbol{\rho}'-\boldsymbol{\rho}|^{3/2}}e^{ik|\boldsymbol{\rho}'-\rho|}d^2\boldsymbol{\rho}'. \end{aligned} \tag{2.6}$$

The formulas (2.2a) and (2.2b) are vector analogues of a well-known Rayleigh diffraction integral [11] for the outgoing solution of the Dirichlet's boundary value problem for the scalar Helmholtz equation in the half-space $z>0$. Moreover, since the boundary values of the z-component E_z of the electric field on the plane $z=0$ are also known [given by Eq. (2.6)] and since E_z also satisfies the Helmholtz equation throughout the half-space, $z>0$ and behaves as an outgoing wave in that half-space it can also be calculated from the Rayleigh diffraction integral.

3. The spectral degree of coherence of the electric field.

We will follow Zernike's precedent of identifying the degree of coherence of the field at two points $Q_1(\boldsymbol{r_1})$ and $Q_2(\boldsymbol{r_2})$ as being a measure of the sharpness of interference fringes which would be formed by allowing the fields from the immediate neighborhood of these points to superpose. Thus we have to consider the distribution of the average intensity, more precisely of the averaged electric energy density in the interference pattern formed in Young's interference experiment, with pinholes at the points Q_1 and Q_2 in an opaque screen placed across the field (see Fig. 1).

Let dA_1 and dA_2 be the areas of the two pinholes. As we noted at the end of Section 2, the Cartesian components of the electric field at a point $P(\boldsymbol{r})$ in the Young interference pattern are given by Rayleigh's formula, i.e.

$$E_\alpha(\boldsymbol{r},\omega) = -\frac{1}{2\pi}\int_{A_1+A_2} E_\alpha(\boldsymbol{\rho},\omega)\, G_z(\boldsymbol{r},\boldsymbol{\rho})\, d^2\boldsymbol{\rho}, \qquad (\alpha = x, y, z), \tag{3.1}$$

where integration extends over the two small apertures A_1 and A_2. Assuming that the apertures are sufficiently small, the expression (3.1) may be approximated by the formula

$$E_\alpha(\mathbf{r},\omega) = -\frac{1}{2\pi}\left[E_\alpha(\boldsymbol{\rho}_1,\omega)G_z(\mathbf{r},\boldsymbol{\rho}_1)dA_1 + E_\alpha(\boldsymbol{\rho}_2,\omega)G_z(\mathbf{r},\boldsymbol{\rho}_2)dA_2\right], \qquad (\alpha = x, y, z). \tag{3.2}$$

The average electric energy density at the point $P(\mathbf{r})$ in the observation plane is given by the trace of the 3×3 electric correlation matrix (2.1), evaluated for $\mathbf{r}_1 = \mathbf{r}_2 = \mathbf{r}$, i.e. it is given by the by the formula

$$\begin{aligned} S(\mathbf{r},\omega) &= Tr\,\overleftrightarrow{W}(\mathbf{r},\mathbf{r},\omega) \\ &= S_x(\mathbf{r},\omega) + S_y(\mathbf{r},\omega) + S_z(\mathbf{r},\omega), \end{aligned} \tag{3.3}$$

where

$$S_\alpha(\mathbf{r},\omega) = \left\langle E_\alpha^*(\mathbf{r},\omega)E_\alpha(\mathbf{r},\omega)\right\rangle, \qquad (\alpha = x, y, z), \tag{3.4}$$

with $E_\alpha(\mathbf{r},\omega)$ being given by the expression (3.2). The z-derivative of the Green's function (2.3) which appear in Eq. (3.2) is given by the expression

$$G_z(\mathbf{r},\boldsymbol{\rho}) = z\left(\frac{ik}{R} - \frac{1}{R^2}\right)\frac{e^{ikR}}{R}, \tag{3.5}$$

where

$$R = |\boldsymbol{\rho} - \mathbf{r}|. \tag{3.6}$$

Assuming that the distance R is much greater than the wavelength $\lambda = 2\pi c/\omega$, G_z may be approximated by the expression

$$G_z(\mathbf{r},\boldsymbol{\rho}) \approx ik\left(\frac{z}{R}\right)\frac{e^{ikR}}{R}. \tag{3.7}$$

It follows that the electric field components, given by Eqs. (3.2), may be expressed in the form

$$E_\alpha(\boldsymbol{r},\omega) = -\frac{i}{\lambda}\left[E_\alpha^{(1)}(\boldsymbol{\rho_1};\omega)\left(\frac{z}{R_1}\right)\frac{e^{ikR_1}}{R_1}dA_1 + E_{\boldsymbol{\alpha}}^{(2)}(\boldsymbol{\rho_2};\omega)\left(\frac{z}{R_2}\right)\frac{e^{ikR_2}}{R_2}dA_2\right], \quad (\alpha = x, y, z). \tag{3.8}$$

On substituting from Eq. (3.8) into Eq. (3.4) we obtain for the spectral density $S_\alpha(\boldsymbol{r},\omega)$ of the electric field component $E_\alpha(\boldsymbol{r};\omega)$ the expression

$$\begin{aligned} S_\alpha(\boldsymbol{r};\omega) = \frac{1}{\lambda^2}\Bigg[& S_\alpha(\boldsymbol{\rho}_1;\omega)\left(\frac{z}{R_1}\right)^2\frac{(dA_1)^2}{R_1^2} + S_\alpha(\boldsymbol{\rho}_2;\omega)\left(\frac{z}{R_2}\right)^2\frac{(dA_2)^2}{R_2^2} \\ & + 2\,\mathrm{Re}\left[W_{\alpha\alpha}(\boldsymbol{\rho_1},\boldsymbol{\rho_2};\omega)\frac{e^{ik(R_2-R_1)}}{R_1R_2}\left(\frac{z^2}{R_1R_2}\right)\right]dA_1dA_2\Bigg], \qquad (\alpha = x, y, z), \end{aligned} \tag{3.9}$$

where Re denotes the real part and the asterisk denotes the complex conjugate. On substituting from Eq. (3.9) into Eq. (3.3) we obtain for the average electric energy density at a point $P(\boldsymbol{r})$ in the interference pattern the expression

$$\begin{aligned} S(\boldsymbol{r};\omega) = \frac{1}{\lambda^2}\Bigg[& S(\boldsymbol{\rho}_1;\omega)\left(\frac{z}{R_1}\right)^2\frac{(dA_1)^2}{R_1^2} + S(\boldsymbol{\rho}_2;\omega)\left(\frac{z}{R_2}\right)^2\frac{(dA_2)^2}{R_2^2} + \\ & + 2\,\mathrm{Re}\left[Tr\,\overset{\leftrightarrow}{W}(\boldsymbol{\rho_1},\boldsymbol{\rho_2};\omega)\frac{e^{ik(R_2-R_1)}}{R_1R_2}\left(\frac{z^2}{R_1R_2}\right)\right]dA_1dA_2\Bigg], \end{aligned} \tag{3.10}$$

where *Tr* stands for the trace of the W-matrix. We may re-write the formula (3.10) in a physically more significant form by noting that the first two terms on the right-hand side of Eq. (3.10) have a clear meaning. The first term represents the average electric energy density

$$S^{(1)}(\boldsymbol{r};\omega) = \frac{1}{\lambda^2}S(\boldsymbol{\rho}_1;\omega)\left(\frac{z}{R_1}\right)^2\frac{(dA_1)^2}{R_1^2} \tag{3.11}$$

when the field reaches the point of observation $P(\boldsymbol{r})$ only through the first pinhole, i.e. when $dA_2 = 0$. Similarly the second term represents the average electric spectral density

$$S^{(2)}(\boldsymbol{r};\omega) = \frac{1}{\lambda^2} S(\boldsymbol{\rho}_2;\omega)\left(\frac{z}{R_2}\right)^2 \frac{(dA_2)^2}{R_2^2} \tag{3.12}$$

when the field reaches the point of observation only through the second pinhole. Hence the formula (3.10) can be expressed in the form

$$S(\boldsymbol{r},\omega) = S^{(1)}(\boldsymbol{r},\omega) + S^{(2)}(\boldsymbol{r},\omega) + 2\sqrt{S^{(1)}(\boldsymbol{r},\omega)}\sqrt{S^{(2)}(\boldsymbol{r},\omega)}\,\mathrm{Re}\left[\eta(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)\frac{e^{ik(R_2-R_1)}}{R_1 R_2}\right], \tag{3.13}$$

where

$$\eta(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) = \frac{Tr\,\overset{\leftrightarrow}{W}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)}{\sqrt{S(\boldsymbol{\rho}_1,\omega)}\sqrt{S(\boldsymbol{\rho}_2,\omega)}} \tag{3.14a}$$

$$\equiv \frac{Tr\,\overset{\leftrightarrow}{W}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)}{\sqrt{Tr\,\overset{\leftrightarrow}{W}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_1,\omega)}\sqrt{Tr\,\overset{\leftrightarrow}{W}(\boldsymbol{\rho}_2,\boldsymbol{\rho}_2,\omega)}}. \tag{3.14b}$$

The formula (3.13) expresses a general spectral interference law for random statistically stationary electromagnetic fields. We see that as the path difference $R_2 - R_1$ varies, the average electric energy density $S(\boldsymbol{r},\omega)$ in the neighborhood of the point of observation $P(\boldsymbol{r})$ varies sinusoidally between the values

$$S_{\max}(\boldsymbol{r},\omega) = S^{(1)}(\boldsymbol{r},\omega) + S^{(2)}(\boldsymbol{r},\omega) + 2\sqrt{S^{(1)}(\boldsymbol{r},\omega)}\sqrt{S^{(2)}(\boldsymbol{r},\omega)}\left|\eta(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)\right|, \tag{3.15a}$$

$$S_{\min}(\boldsymbol{r},\omega) = S^{(1)}(\boldsymbol{r},\omega) + S^{(2)}(\boldsymbol{r},\omega) - 2\sqrt{S^{(1)}(\boldsymbol{r},\omega)}\sqrt{S^{(2)}(\boldsymbol{r},\omega)}\left|\eta(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)\right|. \tag{3.15b}$$

Hence, the spectral visibility of the fringes in the neighborhood of the point $P(\boldsymbol{r})$ in the interference pattern is given by the expression

$$V(\boldsymbol{r},\omega) \equiv \frac{S_{\max}(\boldsymbol{r},\omega) - S_{\min}(\boldsymbol{r},\omega)}{S_{\max}(\boldsymbol{r},\omega) + S_{\min}(\boldsymbol{r},\omega)}$$

$$= \frac{2\sqrt{S^{(1)}(\boldsymbol{r},\omega)}\sqrt{S^{(2)}(\boldsymbol{r},\omega)}}{S^{(1)}(\boldsymbol{r},\omega) + S^{(2)}(\boldsymbol{r},\omega)} \left|\eta(\boldsymbol{\rho_1},\boldsymbol{\rho}_2,\omega)\right|. \tag{3.16}$$

If, as is usually the case, $S^{(1)}(\boldsymbol{r},\omega) = S^{(2)}(\boldsymbol{r},\omega)$ the formula (3.16) reduces to

$$V(\boldsymbol{r},\omega) \equiv \left|\eta(\boldsymbol{\rho_1},\boldsymbol{\rho}_2,\omega)\right|, \tag{3.17}$$

i.e. the absolute value of the expression (3.14) is equal to the spectral visibility of the interference fringes. Hence, if following Zernike, we interpret the absolute value of the degree of coherence of the field at points Q_1 and Q_2 as the visibility of fringes which would be formed by the field reaching the point of observation from the two pinholes, we see that the modulus of the expressions (3.14) represents the absolute value of the degree of coherence[1]. The phase of η is associated with the location of the maxima in the fringe pattern and may also be measured (see [13] and [14]). Hence we may identify $\eta(\boldsymbol{r_1},\boldsymbol{r_2},\omega)$, given by the formulas (3.14), with the (generally complex) electric spectral degree of coherence at frequency ω of any statistically stationary electromagnetic field which propagates into the half-space $z > 0$. The formulas (3.14) are a natural generalization of the expression for the spectral degree of coherence introduced not long ago for beam like fields [7][2].

4. An example.

We will illustrate our main result by deriving an expression for the spectral degree of coherence of the electric field emerging from a cavity containing blackbody radiation. We assume that the opening in the cavity is located in the plane $z = 0$ and that its linear dimensions are sufficiently large, so that effects of diffraction from the edges of the aperture may be neglected. The 3×3 electric cross-spectral density matrix of the radiation emerging from the cavity into the half-space $z > 0$ is given by the expression ([12], Eq. (3.10))

$$W_{\alpha\beta}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2; \omega) = \pi A \left\{ \delta_{\alpha\beta}\left[j_0(\mathrm{k}\rho) - \frac{1}{\rho} j_1(\mathrm{k}\,\rho) \right] + \frac{\rho_\alpha \rho_\beta}{\rho^2} j_2(\mathrm{k}\rho) \right\},$$

$$(\alpha = x, y, z, \quad \beta = x, y, z), \tag{4.1}$$

where

$$\boldsymbol{\rho} = \boldsymbol{\rho_2} - \boldsymbol{\rho_1} = (\rho_x, \rho_y, \rho_z), \quad \rho = |\boldsymbol{\rho}|, \tag{4.2}$$

A represents the Planck distribution

$$A = \frac{8\pi h \nu^3}{c^3} \frac{1}{e^{h\nu/KT} - 1}, \tag{4.3}$$

$\nu = \omega/2\pi$ being the angular frequency, c is the speed of light in vacuum, h is the Planck's constant, K is the Boltzmann's constant, T is the absolute temperature and $\delta_{\alpha\beta}$ is the Kronecker symbol. Further j_0, j_1 and j_2 are the spherical Bessel functions of the first kind and of orders 1, 2 and 3 respectively:

$$j_0(\mathrm{k}\rho) = \frac{\sin(\mathrm{k}\rho)}{\mathrm{k}\rho}, \tag{4.4a}$$

$$j_1(\mathrm{k}\rho) = \frac{\sin(\mathrm{k}\rho)}{(\mathrm{k}\rho)^2} - \frac{\cos(\mathrm{k}\rho)}{\mathrm{k}\rho}, \tag{4.4b}$$

and

$$j_2(\mathrm{k}\rho) = \left[\frac{3}{(\mathrm{k}\rho)^3} - \frac{1}{\mathrm{k}\rho}\right]\sin(\mathrm{k}\rho) - \frac{3}{(\mathrm{k}\rho)^2}\cos(\mathrm{k}\rho). \tag{4.4c}$$

It follows at once from Eq. (4.1) and from the expressions (4.4) for the spherical Bessel functions that the diagonal elements of the 3×3 electric cross-spectral density matrix in the opening are

$$W_{\alpha\alpha}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) = \pi A\left[j_0(\mathrm{k}\rho) - \frac{1}{\mathrm{k}\rho} j_1(\mathrm{k}\rho) + \frac{\rho_\alpha^2}{\rho^2} j_2(\mathrm{k}\rho)\right], \qquad (\alpha = x, y, z). \tag{4.5}$$

Hence,

$$\begin{aligned} Tr\overset{\leftrightarrow}{W}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) &= \pi A\left[3j_0(\mathrm{k}\rho) - \frac{3}{(\mathrm{k}\rho)} j_1(\mathrm{k}\rho) + j_2(\mathrm{k}\rho)\right] \\ &= 2\pi A\frac{\sin(\mathrm{k}\rho)}{\mathrm{k}\rho} \end{aligned} \tag{4.6}$$

and, consequently,

$$S(\omega) \equiv Tr\overset{\leftrightarrow}{W}(\boldsymbol{\rho}, \boldsymbol{\rho}, \omega) = 2\pi A. \tag{4.7}$$

On substituting from Eqs. (4.6) and (4.7) into Eq. (3.14a) and recalling Eq. (4.2) we obtain the following expression for the spectral degree of coherence of the electric field in a large opening of the cavity containing blackbody radiation:

$$\eta(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2; \omega) = \frac{\sin[k|\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1|]}{k|\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1|}. \tag{4.8}$$

References

[1] F. Zernike, The concept of degree of coherence and its application to optical problems, *Physica*, **5**, 785 (1938).

[2] F. Zernike, "Diffraction and optical image formation", *Proc. Phys. Soc. (London)*, **61**, 158-164 (1948).

[3] E. Wolf, "A macroscopic theory of interference and diffraction of light from finite sources II. Fields with a spectral range of arbitrary width", *Proc. Roy. Soc. A*, **230**, 246 (1955).

[4] B. Karczewski, "Coherence theory of the electromagnetic field", *Il Nuovo Cimento,* X, **30**, 906-915 (1963).

[5] W. H. Carter and E. Wolf, "Far-zone behavior of electromagnetic fields generated by fluctuating current distributions", Phys. Rev. A **36**, 1258-1269 (1987).

[6] L. Mandel and E. Wolf, "Spectral coherence and concept of cross-spectral purity", *J. Opt. Soc. A*, **66**, 529-535 (1976); E. Wolf and W.H. Carter, "Angular distribution of radiant intensity from sources of different degrees of spatial coherence", *Opt. Comm.* **13**(3), 205-209 (1975); E. Wolf and W.H. Carter "A radiometric generalization of the van Cittert-Zernike theorem for fields generated by sources of arbitrary state of coherence", *Opt. Comm.* **16**(3), 297-302 (1976); see also M. J. Bastiaans "A frequency-domain treatment of partial coherence", *Opt. Acta*, **24**, 261-274 (1977).

[7] E. Wolf, "Unified theory of coherence and polarization of statistical electromagnetic beams", *Phys. Lett.* A **312,** 263-267 (2003).

[8] R. K. Luneberg, *Mathematical theory of optics* (University of California Press, Berkeley and Los Angeles, 1964), pp. 319-320.

[9] L. Mandel and E. Wolf, *Optical coherence and quantum optics* (Cambridge University Press, Cambridge, 1995).

[10] B. Karczewski and E. Wolf, "Comparison of three theories of electromagnetic diffraction at an aperture. Part I: Coherence matrices", *J. Opt. Soc. Am.* **56**(9), 1207-1214 (1966).

[11] M. Born and E. Wolf, *Principles of optics*, 7th expanded edition (Cambridge: Cambridge University Press, 1999).

[12] C. L. Mehta and E. Wolf, "Coherence properties of blackbody radiation. III. Cross-spectral tensors", *Phys. Rev.* **161**(5), 1328-1334 (1967).

[13] D. F. V. James and E. Wolf, "Determination of the degree of coherence of light from spectroscopic measurements", *Opt. Comm.* **145**, 1-4 (1998).

[14] V. Nirmal Kumar and D. Narayana Rao, "Two-beam interference experiments in the frequency-domain to measure the complex degree of spectral coherence", *J. Mod. Opt.* **48**(9), 1455-1465 (2001).

Footnotes

1. Expression (3.14) was noted previously ([5], Eq. 6.16) as the degree of coherence in a particular case, namely in the far field generated by three-dimensional fluctuating charge current distribution in free space.
2. An analogous definition of the degree of coherence of a beam-like field in the space-time domain was obtained many years ago by Karczewski in a little known paper [4].

Figure

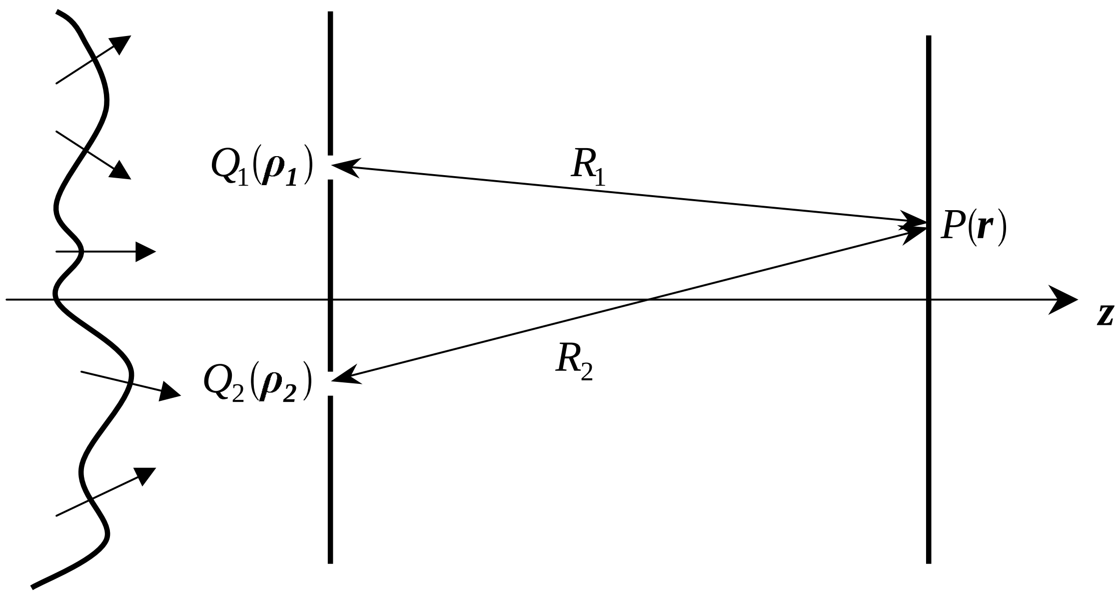

Figure caption

Illustrating the notation relating to determining the spectral degree of coherence of a three-dimensional electromagnetic field from Young's interference experiment.