

SHOW ALL WORK!

In each problem, round all presented results to 4 places after the decimal point.

Problem 1 (35 pts)

A tutoring service has kept records of performance on a standardized test and the number of days students attend their review classes. The performance rating Y represents the percent improvement in the test score students attain after taking the exam a second time. X is the number of attendance days in the review class.

X, Attendance Days	x_0 1	x_1 2.5	x_2 5	x_3 6.5	x_4 9
Y, % Improvement	y_0 2	y_1 5	y_2 11	y_3 14	y_4 17

- a) Estimate $f(4)$, the % improvement in one's score after 4 days of attending review classes, assuming $y = f(x)$ is the true function relating X and Y. Base your result on a third order Newton divided-difference interpolating polynomial $f_3(x)$. Fill in the table of divided differences below to find the appropriate coefficients.

i	x_i	y_i	Δ	Δ^2	Δ^3	Δ^4
0						
1						
2						
3						
4						

- b) Estimate the error in the interpolated value $f(4)$. Use the extra data point. i.e. the one not used in Part a) to obtain your answer.
- c) Fit a power equation to the data.

Problem 2 (35 pts)

Given the following system of equations

$$\begin{array}{rccccrcr} x_1 & + & x_2 & + & x_3 & - & 3x_4 & = & 2 \\ -x_1 & & & & + & x_3 & - & x_4 & = & 1 \\ 2x_1 & - & x_2 & + & 3x_3 & - & 8x_4 & = & 2 \\ 5x_1 & + & x_2 & + & 4x_3 & - & 13x_4 & = & 5 \end{array}$$

- Show the system of equations are consistent by converting the augmented matrix into its echelon form. How many (if any) arbitrary unknowns are there?
- If there are one or more arbitrary unknowns, determine whether x_2 can be arbitrary **without finding the solution.**
- Convert the augmented matrix into its row reduced echelon form.
- Is x_4 arbitrary? Explain.

Problem 3 (30 pts)

Measurements of an object's speed were recorded at uniform increments of time. The results are tabulated below.

t sec	$v(t)$ ft/sec	t sec	$v(t)$ ft/sec
0.0	29.6	5.0	86.9
1.0	42.8	6.0	98.6
2.0	54.5	7.0	109.2
3.0	57.5	8.0	119.5
4.0	76.5	9.0	128.3

- Find the Normal equations used to solve for the coefficients of the Least Squares line. Use the table below to assist with the calculations.
- Find the equation of the Least Squares line.
- Calculate SSE, SST and SSR.
- Find the correlation coefficient r .

t_i	v_i	t_i^2	$t_i v_i$	$\hat{v}_i = a_0 + a_1 t_i$	$e_i = v_i - \hat{v}_i$	e_i^2
0	29.6					
1	42.8					
2	54.5					
3	57.5					
4	76.5					
5	86.9					
6	98.6					
7	109.2					
8	119.5					
9	128.3					
$\sum_{i=1}^n t_i$	$\sum_{i=1}^n v_i$	$\sum_{i=1}^n t_i^2$	$\sum_{i=1}^n t_i v_i$			$\sum_{i=1}^n e_i^2$