

**SHOW ALL WORK!**

Problem 1 (35 pts)

Consider the definite integral

$$I = \int_a^b f(x)dx \quad \text{where} \quad a = 3.1, \quad b = 3.9 \quad \text{and} \quad f(x) = \frac{1}{x}$$

1. Use Simpson's Rule with 8 intervals to approximate I. Fill in the table below with  $f(x_i)$  rounded to 8 places after the decimal point. Express your answer to 8 places after the decimal point.

i	$x_i$	$f(x_i)$
0	3.1	0.32258065
1		
2		
3		
4		
5		
6		
7		
8		

2. Use the Gauss Quadrature two point formula to approximate I. Express your answer to 8 places after the decimal point.

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Problem 2 (35 pts)

The following data points were obtained experimentally.

i	$x_i$	$y_i$				
1	1	1.6667				
2	3	3.7500				
3	5	5.0000				
4	7	5.8333				
5	10	6.6667				
6	50	9.0909				
7	100	9.5238				
8	150	9.6774				
Total						

1. Prepare a rough graph of the data and determine whether an exponential model, a saturation growth model, or a power equation model best describes the data.
2. Based on the appropriate model, transform the data so that a linear model can be fit to the transformed data. Enter the transformed data into the table.
3. Use Least Squares Regression to determine the parameters of the linear model. Fill in the last two columns of the table to find the coefficients in the normal equations. You may use your calculator to solve the normal equations.
4. Find the parameters of the nonlinear model.

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Problem 3 (30 pts)

Given the following system of equations

$$\begin{aligned}x_1 + x_2 + x_3 - 3x_4 &= 2 \\-x_1 + x_3 - x_4 &= 1 \\2x_1 - x_2 + 3x_3 - 8x_4 &= 2 \\x_2 + x_3 - 2x_4 &= 2\end{aligned}$$

1. Convert the augmented matrix into its echelon form and show there are an infinite number of solutions.
2. Determine whether  $x_2$  can be arbitrary **without finding the solution.**
3. Show that  $x_4$  is arbitrary and find the solution with  $x_4$  as the arbitrary unknown.

