EGN 3420 Exam 1 Sp 97

Problem 1 (25 pts)

The function $f(x) = x^4 - 10x^2 + 9$ has a root located between 2 and 5. Fill in the tables below for the first three iterations of the Bisection Method and the False Position method. Express all answers to four digits after the decimal point.

Iteration	x_l	x_u	x_r	$f(x_l)$	$f(x_r)$	e_A
1	2	5				
2						
3						

Bisection Method

Iteration	x_l	x_u	x_r	$f(x_l)$	$f(x_u)$	$f(x_r)$	e_A
1	2	4					
2							
3							

False Position Method

Problem 2 (25 pts)

Use the simple one point iteration method to find the root of

$$f(x) = x - \ln(x^2 + 2) = 0$$

Start with $x_0 = 0$ and fill in the table below. Round all entries in the table to 4 places after the decimal point. Do not round the results of intermediate calculations.

i	x_i	$f(x_i)$
0	0	
1		
2		
5		
10		

Problem 3 (25 pts)

Consider the following system of equations:

$$
\begin{array}{rrrrrrrrrrrrr}
 & & v & + & w & + & x & + & 2y & & & = & 4 \\
u & + & v & + & w & + & x & - & y & - & z & = & 2 \\
2u & & & - & 3w & + & x & & & + & z & = & 3 \\
-u & & & + & 2w & + & x & + & y & - & z & = & 1 \\
u & - & v & & & & & & & - & 2z & = & 0
\end{array}
$$

A) Transform the augmented matrix (A|b) into its Echelon Form by performing a sequence of elementary row operations. Without solving for the general solution, determine if z can be chosen arbitrarily. Repeat for x and y.

$$
\left[\begin{array}{cccccc|c}
1 & 1 & 1 & 1 & -1 & -1 & 2 \\
0 & 1 & 1 & 1 & 2 & 0 & 4 \\
2 & 0 & -3 & 1 & 0 & 1 & 3 \\
-1 & 0 & 2 & 1 & 1 & -1 & 1 \\
1 & -1 & 0 & 0 & 0 & -2 & 0
\end{array}\right]
$$

Original Augmented Matrix

B) Start with the Echelon Form and use back substitution to find the general solution expressed in terms of one or more arbitrary unknowns.

Problem 4 (25 pts)

Consider the function $f(x) = x^{10} - 1$.

A) Find an expression for the 2nd order truncated Taylor Series Expansion $f_2(x)$ evaluated at $x=1.01x_0$, where x_0 is the point at which the series is expanded about. Leave your simplified answer in terms of x_0. Numeric constants should be rounded to 5 places after the decimal point.

B) If $x_0=1$, find the true relative error, as a per cent, in $f_2(1.01x_0)$.

C) If $x_0=1$, find $f_3(1.01x_0)$.

In Parts B) and C) express your answer rounded to 5 places after the decimal point.