

Show All Work! Use Only Methods Discussed In Class!

Problem 1 (25 pts)

Estimate the integral $I = \int_0^{p/2} \sin x dx$

A) Using Trapezoidal Integration with 10 equally spaced intervals.

B) Using Simpsons 1/3 Rule with 10 equally spaced intervals.

Fill in the following table to help with the calculations. Round all calculations to 4 places after the decimal.

i	x_i	f_i
0	0.0000	0.0000
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

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Problem 2 (25 pts)

The following integral is to be approximated using Trapezoidal Integration.

$$I = \int_0^4 \left[-\frac{1}{12}(x-2)^4 + 2x^2 \right] dx$$

Let I_n represent the approximation to I when the interval $(0,4)$ is subdivided into n equal sub-intervals. How many sub-intervals must there be so that the error $|I - I_n|$ is at most $\frac{10^{-4}}{3}$?

Problem 3 (25 pts)

A quadratic spline is to be fit thru the following points: (0,1), (1,2), (3,3), (4,2).

The spline is given by

$$f(x) = \begin{cases} b_1x + c_1 & 0 \leq x \leq 1 \\ a_2x^2 + b_2x + c_2 & 1 \leq x \leq 3 \\ a_3x^2 + b_3x + c_3 & 3 \leq x \leq 4 \end{cases}$$

A system of 8 equations in the 8 unknowns $b_1, c_1, a_2, b_2, c_2, a_3, b_3, c_3$ can be represented by $A\underline{x} =$

\underline{b} . The vectors are $\underline{x} = [b_1, c_1, a_2, b_2, c_2, a_3, b_3, c_3]^T$ and $\underline{b} = [2, 2, 3, 3, 1, 2, 0, 0]^T$

Find the coefficient matrix A.

Problem 4 (25 pts)

An unknown function generated the following data points:

(0,0), (1,1.6487), (3,13.4451)

- A) Find the Newton Divided Difference interpolating polynomial $f_2(x)$.
- B) An additional data point (2,5.4366) is available. Estimate the error in $f_2(2.5)$.
- C) The data points were generated from the function $f(x) = xe^{x/2}$. Find the true error in $f_2(2.5)$.
- D) Estimate of error in $f_2(2.5) =$ _____
- E) True error in $f_2(2.5) =$ _____

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Problem 5 (25 pts)

Solve the following system of equations by using a sequence of elementary row operations on the augmented matrix until it has been reduced to its Echelon form.

$$\begin{array}{rcccccccl} w & + & x & + & y & - & z & = & 1 \\ 2w & - & x & + & 3y & - & 2z & = & -2 \\ w & & & + & y & - & z & = & 0 \\ & & 2x & + & y & & & & 1 \end{array}$$