

Show All Work!

Problem 1 (35 pts)

A retail store owner is trying to determine the best price for a popular tee-shirt. He has been experimenting with different prices to assess its effect on sales. The following sales records have been accumulated over the past six weeks.

Week No.	1	2	3	4	5	6
Shirt Price (p)	\$3.00	\$6.00	\$5.25	\$3.75	\$3.50	\$4.75
Weekly Sales (s)	855	296	403	738	804	512

- a) Find the coefficients a_0, a_1, \dots, a_5 in the standard form of the interpolating polynomial

$$s_5(p) = a_0 + a_1p + a_2p^2 + a_3p^3 + a_4p^4 + a_5p^5$$

by solving the 6 x 6 system of equations, $A\underline{x} = \underline{b}$ where \underline{x} is the vector $[a_0 \ a_1 \ a_2 \ a_3 \ a_4 \ a_5]^T$. You can use your calculator to find the solution. Express all answers in scientific notation with 4 places after the decimal point in the mantissa.

- b) Estimate the amount of weekly sales if the shirts are priced at \$5.00 each.
- c) Use the result of Part a) to estimate the weekly revenue r (Weekly Sales x Shirt Price) if the shirts are priced at \$4.00 each.

Show All Work!

Problem 2 (35 pts)

The table below lists the target heart rate during exercise for different ages.

Age (x)	20	25	30	35	40	45	50	55	60	65	70
Target Heart Rate (y)	150	146	142	140	135	131	127	124	120	116	112

- a) Use a 3rd order Newton Divided Difference interpolating polynomial to estimate the target heart rate of a 44 year old person. Complete the table below with the appropriate data points selected and fill in all the relevant divided differences necessary to calculate b_0 , b_1 , b_2 , and b_3 in

$$y_3(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) + b_3(x - x_0)(x - x_1)(x - x_2)$$

To avoid round-off errors, express all irrational values in the table as fractions. For example, 0.8 is not irrational but $0.1/15$ is and should be converted to $1/150$ for all subsequent calculations.

Age (x)	Target Heart Rate (y)	$f_1[]$	$f_2[]$	$f_3[]$	$f_4[]$
35	140				
40	135				
45	131				
50	127				
60	120				

- b) Use the additional data point (60, 120) to estimate the error $R_3(44)$ in $y_3(44)$ from Part a).

Show All Work!

Problem 3 (30 pts)

A quadratic spline is to be fit through the following points: (0,1), (2,4), (3,5), (6,9). The spline is given by

$$f(x) = \begin{cases} a_1 + b_1x & 0 \leq x \leq 2 \\ a_2 + b_2x + c_2x^2 & 2 \leq x \leq 3 \\ a_3 + b_3x + c_3x^2 & 3 \leq x \leq 6 \end{cases}$$

A system of 8 equations in 8 unknowns has been generated, i.e. $A\underline{x} = \underline{b}$. The vectors \underline{x} and \underline{b} are

$$\underline{x} = \begin{bmatrix} a_1 \\ b_1 \\ a_2 \\ b_2 \\ c_2 \\ a_3 \\ b_3 \\ c_3 \end{bmatrix} \quad \text{and} \quad \underline{b} = \begin{bmatrix} 4 \\ 4 \\ 5 \\ 5 \\ 1 \\ 9 \\ 0 \\ 0 \end{bmatrix}$$

Find the matrix A .