

SHOW ALL WORK!

Problem 1 (25 pts)

Consider the definite integral $I = \int_0^{1.6} 10x^9 dx$

Fill in the table below with the integrand $f(x_i)$ rounded to 6 places after the decimal point.

i	x_i	$f(x_i)$
0		
1		
2		
3		
4		
5		
6		
7		
8		

- A) Use Trapezoidal Integration with 4 intervals to approximate I. Express your answer to 6 places after the decimal point.
- B) Use Trapezoidal Integration with 8 intervals to approximate I. Express your answer to 6 places after the decimal point.
- C) Use the results of Parts A) and B) to approximate I. Express your answer to 6 places after the decimal point.
- D) Find the per cent true relative error in Parts A), B) and C). Express your answer to 2 places after the decimal point.

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Problem 2 (20 pts)

Consider the definite integral $I = \int_1^2 5x^4 dx$

- A) Use the Gauss Quadrature two point formula to approximate I. Express your answer to 6 places after the decimal point.
- B) Find the per cent true relative error in Part A). Express your answer to 6 places after the decimal point.

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Problem 3 (25 pts)

Given the following system of equations

$$\begin{array}{rcccccccc} x_1 & + & x_2 & + & x_3 & - & 3x_4 & = & 6 \\ -x_1 & + & x_2 & + & x_3 & - & x_4 & = & 0 \\ 2x_1 & - & x_2 & + & 3x_3 & - & 8x_4 & = & 3 \\ & & x_2 & + & x_3 & - & x_4 & = & 4 \end{array}$$

- A) Determine if the system of equations above is consistent or inconsistent.
- B) If the system is consistent, determine if there is a unique solution or an infinite number of solutions.
- C) If there is a unique solution, use the Gauss-Jordan Method to find the solution.

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Problem 4 (30 pts)

The field strength of the earth's gravitational field g diminishes with distance h from the surface of the earth. The following table shows the field strength at several distances.

Distance, h (10^6 m)	Field Strength, g (N/kg)	Distance, h (10^6 m)	Field Strength, g (N/kg)
0	9.81	20	0.58
2.5	5.07	30	0.30
5	3.09	40	0.19
10	1.49	50	0.12

An interpolating polynomial is required to estimate g at 15×10^6 m.

- A) Find the Newton Divided Difference Interpolating polynomial $f_3(h)$ to be used for estimating g at 15×10^6 m. Prepare a table of divided differences and express all answers to 8 places after the decimal point.
- B) Estimate g at 15×10^6 m. Round your answer to 2 places after the decimal point.
- C) The true function relating g and h is

$$g = f(h) = \frac{Gm_E}{(R_E + h)^2}$$

where $G = 6.7 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$ (gravitational constant)
 $m_E = 6.0 \times 10^{24} \text{ kg}$ (mass of the earth)
 $R_E = 6.4 \times 10^6 \text{ m}$ (radius of the earth)

- Find the per cent true error in the estimate obtained in Part B). Round your answer to 2 places after the decimal point.
- D) Using the additional point at $h = 30 \times 10^6$, estimate g at 15×10^6 m. Round your answer to 2 places after the decimal point.
- C) Estimate the error in your answer from Part B). Round your answer to 2 places after the decimal point.

