

**EEL 3420**  
**Engineering Analysis**

**Fall 2005**

**Home Work Schedule**

## Schedule of Assignments:

<u>HW#</u>	<u>Week</u> <u>Due</u>	<u>Points</u>	<u>Title</u>
1	9/14/05	1 %	Error Analysis
2	9/28/05	2 %	Root Finding Techniques
3	10/12/05	1 %	Matrices
4	10/26/05	2 %	Systems of Linear Equations
5	11/09/05	2 %	Curve Fitting
6	11/30/05	2 %	Numerical Integration and Differentiation

All homework must include:

- A Cover page with the following information  
Course number (EEL 3420)  
Your name  
Home work number
- Short description of the work done.
- Source code with comments.
- Program output.

## Notes:

- Comments on the source code are to explain what the code does. This will help the grader (TA) as well as your self understand how your code works and what each function does. Each function must have comments indicating exactly what that function does.
- Homework assignments are due at the BEGINING of the class meeting for the day it is due. Late HW receives a 1 time penalty of 10%.
- Homework assignments are to be turned in to me (course instructor, Dr. Gonzalez) or put into my mailbox or under my office door (enr 211).
- You are required to keep a copy of all material incase it gets lost. You must also keep all graded material returned to you to prove it has been turned in on time.

**Homework Assignment #1**  
EEL 3420 Engineering Analysis

**Error Analysis**

1. Use zero through third order Taylor series expansions to predict  $f(3)$  for

$$f(x) = 25x^3 - 6x^2 + 7x - 88$$

using a base point of  $x = 2$ . Compute the percent relative error and the number of significant digits for each approximation.

2. Use zero through fourth order Taylor series expansions to predict  $f(3)$  for

$$f(x) = e^{-x}$$

using a base point of  $x = 1$ . Compute the percent relative error and the number of significant digits for each approximation.

3. Recall that the velocity of a falling parachutist can be computed by

$$v = \frac{gm}{c} \left(1 - e^{-\frac{c}{m}t}\right)$$

Use a first-order error analysis to estimate the error of  $v$  at

$$t = 7$$

$$g = 9.8$$

$$m = 68.1 \pm 0.5$$

$$c = 12.5 \pm 2$$

4. Let  $x = 37.5678$  and  $y = 37.5318$ . If our computer can only store 4 digits, what is the relative error for  $x$  and  $y$  when we represent the numbers using our computer. Compute  $z = (x - y) + 0.01946$ . What is the relative error of  $z$ . How many digits of significance does  $z$  have.

5. What is the rate of convergence for  $f(n)$ .  $f(n)$  converges to 0 but at what rate  $O(?)$ . Recall  $\alpha_n$  converges to  $\alpha$  with rate  $\beta_n$  if  $|\alpha_n - \alpha| \leq |\beta_n| \cdot k$  for some constant  $k$

$$f(n) := \frac{(n-2) \cdot (n+4)}{n^3}$$

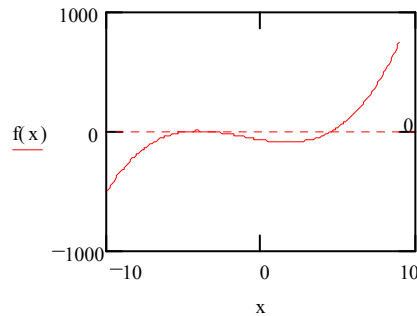
**Homework Assignment #2**  
EEL 3420 Engineering Analysis

**Root Finding Techniques**

1. Find the positive root of the function

$$f(x) := x^3 + 3.5x^2 - 21.0x - 67.5$$

Use a bracket method first to find an initial approximation then use Newton's method to further improve your estimate to 2 digits of accuracy.



2. Find the root of the function

$$f(x) = \frac{1}{x-5} - 5$$

3. Find the value of  $\sqrt{3}$  to 4 digits of accuracy.
4. Using a computer language of your choice implement Newton's method to find the roots of arbitrary polynomials to 4 significant digits. The function will have as its inputs the coefficients of the input function and its degree, the coefficients of the derivative function and an initial guess of the root. In each iteration, the function is to compute the values of  $f(p_k)$  and  $f'(p_k)$  using Horner's method as discussed in class. Recall that Newton's method uses

$$p_{k+1} = p_k - \frac{f(p_k)}{f'(p_k)}$$

Horner's method: The value of the polynomial

$$p(x) = a_0x^n + a_1x^{n-1} + \dots + a_n$$

evaluated at  $x = c$  may be found by using Horner's rule:

$$p(x) = (((a_0c + a_1)c + a_2)c + a_3)c + \dots$$

Using the following algorithm:

```
Ans = A[0];  
For (x = 0; x < N; x++)  
    Ans = Ans *c + A[x];
```

**Homework Assignment #3**  
EEL 3420 Engineering Analysis

**Matrices**

1. Given

$$A = \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$$

show that

$$(A + B)(A - B) \neq A^2 - B^2$$

2. Find all matrices A such that

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \cdot A = A \cdot \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \quad \begin{bmatrix} 3 & 0 \\ 5 & 0 \end{bmatrix} \cdot A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

3. Find the determinant of the two matrices

$$\begin{bmatrix} -3 & 1 & 1 \\ 1 & -3 & 1 \\ 1 & 1 & -3 \end{bmatrix} \quad \begin{bmatrix} 2 & 3 & 1 \\ 1 & 4 & -3 \\ -1 & 2 & 0 \end{bmatrix}$$

4. Prove that

$$|AB| = |A||B|$$

for any 2 square matrices. Assume the matrices are 2x2. Note this is true for all square matrices of size nxn.

5. Find the multiplicative inverse, if it exist, of A where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 5 & 12 \end{bmatrix}$$

6. Find the left multiplicative inverse, if it exist, of

$$\begin{bmatrix} 1 & 1 \\ 3 & 4 \\ 0 & 0 \end{bmatrix}$$

Show that the right inverse does not exist.

7. Determine the rank of the following matrix.

$$\begin{bmatrix} 0 & 2 & 4 & 6 \\ 3 & -1 & 4 & -2 \\ 6 & -1 & 10 & -1 \end{bmatrix}$$

Use the 3 elementary row transformations to simplify the work.

8. Show that the following 3 equations are linearly dependant:

$$2x + z$$

$$x + y$$

$$2y - z$$

9. Find the characteristic equation, the eigenvalues, and a set of eigenvectors for the given 4 matrices.

$$\begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 0 & 2 \\ 0 & 1 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

10. Use the Hamilton-Cayley theorem to find

$$A^{-3}$$

for the matrix

$$A = \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$$

**Homework Assignment #4**  
EEL 3420 Engineering Analysis

**Systems of Equations**

1. Solve, by hand, the following linear systems using Gaussian-elimination.

$$\begin{aligned}x_1 + x_2 - x_3 &= 3 \\2x_1 - x_2 - 3x_3 &= 0 \\-x_1 - 2x_2 + x_3 &= -5\end{aligned}$$

2. Write a MATLAB program to solve the following linear systems using all three Gaussian algorithms and the Gauss-Jordan algorithm.

$$\begin{array}{lll}x_1 + 2x_2 + 3x_3 = 1 & 2x_1 + 4x_2 - x_3 = -5 & .832x_1 + .448x_2 + .193x_3 = 1.00 \\2x_1 + 3x_2 + 4x_3 = -1 & x_1 + x_2 - 3x_3 = -9 & .784x_1 + .421x_2 - .207x_3 = 0.00 \\3x_1 + 4x_2 + 6x_3 = 2 & 4x_1 + x_2 + 2x_3 = 9 & .784x_1 - .421x_2 + 279x_3 = 0.00\end{array}$$

3. Use the Gaussian-Jordan method, by hand, to find the inverse of the following matrix.

$$\begin{bmatrix} 10 & -3 & 6 \\ 1 & 8 & -2 \\ -2 & 4 & -9 \end{bmatrix}$$

4. Modify the MATLAB Gaussian-Jordan program that you implemented so that it can be used to find the inverse of the matrix in the previous problem.
5. Find the fixed points for the following system of nonlinear equations. You may use MATLAB as a tool for the matrix manipulation.

$$\begin{aligned}x &= g_1(x, y, z) = 9x^2 + 36y^2 + 4z^2 - 36 \\y &= g_2(x, y, z) = x^2 - 2y^2 - 20z \\z &= g_3(x, y, z) = 16x - x^3 - 2y^2 - 16z^2\end{aligned}$$

**Homework Assignment #5**  
EEL 3420 Engineering Analysis

**Curve fitting**

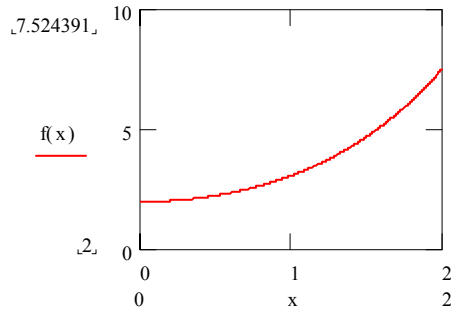
1. Given the following 3 sets of data, for each set, use regression to find a polynomial that best fits the data. For each set determine what percentage of the variation can be explained for. Try 1<sup>st</sup> order, 2<sup>nd</sup> order ... until you get a good fit. Hint: put the data into two matrices so that you can easily manipulate them using MATLAB and generate

$$\sum_{i=1}^n x_i \quad \sum_{i=1}^n (x_i)^2$$

and other terms.

y	g(y)	y	f(y)	y	g2(y)
-10	-351.962	-10	-355	-10	148.038
-9	-242.464	-9	-237.1	-9	85.586
-8	-139.437	-8	-145.4	-8	65.363
-7	-81.543	-7	-76.9	-7	38.507
-6	-26.771	-6	-28.6	-6	38.029
-5	4.074	-5	2.5	-5	35.324
-4	14.929	-4	19.4	-4	27.729
-3	31.028	-3	25.1	-3	35.078
-2	17.122	-2	22.6	-2	17.922
-1	18.164	-1	14.9	-1	18.214
0	5	0	5	0	5
1	-7.364	1	-4.1	1	-7.314
2	-3.922	2	-9.4	2	-3.122
3	-13.828	3	-7.9	3	-9.778
4	7.871	4	3.4	4	20.671
5	25.926	5	27.5	5	57.176
6	65.571	6	67.4	6	130.371
7	130.743	7	126.1	7	250.793
8	200.637	8	206.6	8	405.437
9	317.264	9	311.9	9	645.314
10	441.962	10	445	10	941.962

2. For the set of data points, find an interpolating polynomial that fits the data. You may use either Newton's or Lagrange interpolating polynomial. Do this by hand. Estimate the value at  $x = 1.5$  and find the error.



$$f(x) := e^x + e^{-x}$$

x	f(x)
0	2
1	3.086
2	7.524

3. Write a program to generate 10 evenly spaced points from 0 to 2 using  $f(x)$  above and find a 9<sup>th</sup> order interpolating polynomial using these 10 points. Hint: Newton's algorithm produces the b's. Use this polynomial to estimate the value at  $x = 1.5$  and find the error.

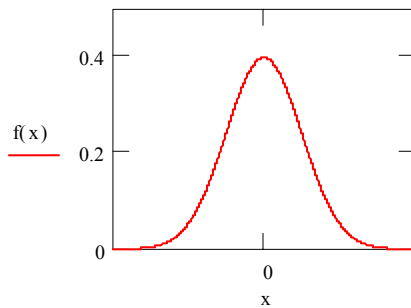
**Homework Assignment #6**  
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**Numerical Integration, Differentiation, and Differential Equations**

- The following function corresponds to the normal distribution. There does not exist a closed form expression of the integral for this function so one must use numerical integration to find the probabilities. Use Simpson's 1/3 and 3/8 rule to integrate  $f(x)$  from 0 to 3. Partition the integrating interval into 10 evenly spaced segments and use the multiple application formula. You may use a computer if you like. Show all work or show the program listing.

$$\mu := 0 \quad \sigma := 1$$

$$f(x) := \frac{1}{\sqrt{2 \cdot \pi \cdot \sigma^2}} \cdot e^{-\frac{(x-\mu)^2}{2 \cdot \sigma^2}}$$



Note

$$\int_0^3 f(x) dx = 0.499$$

- Use Romberg integration to evaluate  $\int_0^4 x e^{2x} dx$  to an accuracy of 0.1%. Present your results as in the notes (showing the Richardson Extrapolations).
- For the function  $y = \sin x$  compute the first derivative at  $x = \frac{\pi}{3}$  using the forward difference approximation of  $O(h^2)$  and the centered difference approximations of  $O(h^4)$  and using a value of  $h = \frac{\pi}{12}$ .
- The following data was collected for the distance traveled versus time for a rocket.

T	0	1	2	3	4	5
Y	0	2.1	7.8	18.2	31.9	50.3

Use numerical differentiation to estimate the rocket's speed and acceleration at time  $T = 3$ .