EEL 3420
Engineering Analysis

Spring 2006

Home Work Schedule

Schedule of Assignments:

HW#	Week Due	Points	Title
1	1/31/05	2 %	Error Analysis
2	2/14/05	2 %	Root Finding Techniques
3	3/07/05	2 %	Matrices
4	4/06/05	2 %	Systems of Linear Equations
5	4/24/05	2 %	Curve Fitting (added problem #4)
6		0 %	Cancelled (Numerical Integration and Differentiation)

All homework must include:

- A Cover page with the following information
  - Course number (EEL 3420)
  - Your name
  - Home work number
- Short description of the work done.
- Source code with comments.
- Program output.

Notes:

- Comments on the source code are to explain what the code does. This will help the grader (TA) as well as your self understand how your code works and what each function does. Each function must have comments indicating exactly what that function does.

- Homework assignments are due at the BEGINING of the class meeting for the day it is due. Late HW receives a penalty of 10%. No HW will be accepted after the next HW is due (approximately 2 weeks). I will not accept any HW after 4/18/2006.

- Homework assignments are to be turned in to me (course instructor, Dr. Gonzalez) or put into my mailbox or under my office door (engr 211).

- You are <u>required to keep a copy of all material</u> incase it gets lost. You must also keep all graded material returned to you to prove it has been turned in on time.

Homework Assignment #1
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Error Analysis

1. Use zero through second order Taylor series expansions to predict $f(1)$ for

$$f(x) = 36x^2 + 5x - 103$$

   using a base point of x = 2. Compute the percent relative error and the number of significant digits for each approximation.

2. Use zero through fourth order Taylor series expansions to predict $f(2.5)$ for

$$f(x) = \ln(x)$$

   using a base point of x = 3/2. Compute the percent relative error and the number of significant digits for each approximation.

3. The Stefan-Boltzmann law is as follows

$$H = Ae\sigma T^4$$

   Use a first-order error analysis to estimate the error of H using

$$A = 0.15 \pm 0.01$$
$$e = 0.9 \pm 0.05$$
$$T = 550 \pm 20$$
$$\sigma = 5.67X10^{-8}$$

4. What is the rate of convergence for f(n). $f(n) \to 0$ as $n \to \infty$ but at what rate O(?). Recall α_n converges to α with rate β_n if $|\alpha_n - \alpha| \le |\beta_n| \cdot k$ for some constant k

$$f(n) = n^2 \cos\left(\frac{1}{n}\right)\frac{(n+2)(n-2)}{n^6}$$

Homework Assignment #2
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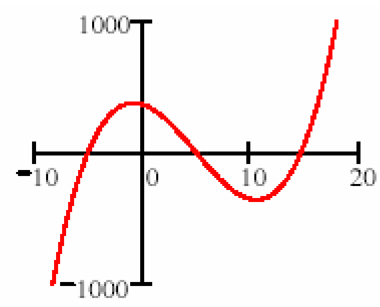
Root Finding Techniques

1. Find the positive root of the function

$$f(x) = x^3 - 14.67x^2 - 25x + 366.75$$

Use a bracket method first to find an initial approximation then use Newton's method to further improve your estimate to 4 digits of accuracy.



2. Find the root of the function

$$f(x) = \frac{1}{x-5} - 5.$$

3. Find the value of $\sqrt{5}$ to 4 digits of accuracy.

4. Using a computer language of your choice implement Newton's method to find the roots of arbitrary polynomials to 4 significant digits. The function will have as its inputs the coefficients of the input function and its degree, the coefficients of the derivative function and an initial guess of the root. In each iteration, the function is to compute the values of $f(p_k)$ and $f'(p_k)$ using Horner's method as discussed in class. Recall that Newton's method uses

$$p_{k+1} = p_k - \frac{f(p_k)}{f'(p_k)}$$

Horner'method: The value of the polynomial

$$p(x) = a_0x^n + a_1x^{n-1} + \cdots + a_n$$

evaluated at x = c may be found by using Horner's rule:

$$p(x) = (((a_0c + a_1)c + a_2)c + a_3)c + \cdots$$

Using the following algorithm:

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Ans = A[0];
For (x = 0; x < N; x++)
    Ans = Ans *c + A[x];
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Homework Assignment #3
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Matrices

1. Show that for any 2 x 2 matrix,

$$A(B+C) = AB + AC$$

In fact this holds for matrices of any dimension provided they are conformable to multiplication

2. For $X = [1 \quad -1 \quad 2]$ compute XX^T and $X^T X$

3. Find all matrices A such that

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} = A \begin{bmatrix} 1 & 5 \\ 10 & 2 \end{bmatrix}$$

4. Find the determinant of the two matrices

$$A = \begin{bmatrix} 2 & 0 & 6 \\ -1 & 5 & -4 \\ 3 & -5 & 2 \end{bmatrix} \text{ and } A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 7 \end{bmatrix}$$

5. Prove that

$$\begin{vmatrix} a & b & m & n \\ c & d & r & s \\ 0 & 0 & e & f \\ 0 & 0 & g & h \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \begin{vmatrix} e & f \\ g & h \end{vmatrix}$$

6. Find the multiplicative inverse, if it exist, of the following matrices

$$A = \begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix} \quad B = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \quad C = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 5 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

7. Determine the rank of the following matrix.

$$A = \begin{bmatrix} 4 & 2 & -1 & 3 \\ 0 & 5 & -1 & 2 \\ 12 & -4 & -1 & 5 \end{bmatrix}$$

Use the 3 elementary row transformations to simplify the work.

8. Are the following 3 equations linearly independent? Prove.

$$\begin{array}{rrrr} 4w & +2x & -y & +3z \\ & 5x & -y & +2z \\ 12w & -4x & -y & +5z \end{array}$$

9. Find the characteristic equation, the eigenvalues, and a set of eigenvectors for the given 2 matrices.

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

10. Prove that if an eigenvalue of A is zero than the determinant of A is zero.

11. Use the Hamilton-Cayley theorem to find A^{-3} for the matrix

$$A = \begin{bmatrix} 7 & 2 \\ 5 & 1 \end{bmatrix}$$

Homework Assignment #4

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Systems of Equations

1. Solve, by hand, the following linear systems using Gaussian-elimination.

$$\begin{aligned} x + 3y + z &= 6 \\ 3x - 2y - 8z &= 7 \\ 4x + 5y - 3z &= 17 \end{aligned} \qquad \begin{aligned} 2x - y + z &= 1 \\ x + y - z &= 2 \\ 3x - y + z &= 0 \end{aligned} \qquad \begin{aligned} 2x - 4y + 5z &= 10 \\ 2x - 11y + 10z &= 36 \\ 4x - y + 5z &= -6 \end{aligned}$$

2. Write a MATLAB program to solve the following linear systems using all three Gaussian algorithms and the Gauss-Jordan algorithm.

$$\begin{aligned} x_1 + 2\cdot x_2 + 3\cdot x_3 &= 1 \\ 2\cdot x_1 + 3\cdot x_2 + 4\cdot x_3 &= -1 \\ 3\cdot x_x + 4\cdot x_2 + 6\cdot x_3 &= 2 \end{aligned} \qquad \begin{aligned} 2\cdot x_1 + 4\cdot x_2 - x_3 &= -5 \\ x_1 + x_2 - 3\cdot x_3 &= -9 \\ 4\cdot x_1 + x_2 + 2\cdot x_3 &= 9 \end{aligned} \qquad \begin{aligned} .832\cdot x_1 + .448\cdot x_2 + .193\cdot x_3 &= 1.00 \\ .784\cdot x_1 + .421\cdot x_2 - .207\cdot x_3 &= 0.00 \\ .784\cdot x_1 - .421\cdot x_2 + 279\cdot x_3 &= 0.00 \end{aligned}$$

3. Use the Gaussian-Jordan method, by hand, to find the inverse of the following matrix.

$$\begin{bmatrix} 1 & -2 & 0 \\ -2 & 3 & 1 \\ -5 & 9 & 3 \end{bmatrix}$$

4. Modify the MATLAB Gaussian-Jordan program that you implemented so that it can be used to find the inverse of the matrix in the previous problem.

5. Find the fixed points for the following system of nonlinear equations. You may use MATLAB as a tool for the matrix manipulation.

$$(x-3)^2 + (y-2)^2 = 25$$
$$y + 2 = (x-3)^2$$

Homework Assignment #5
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Curve fitting

1. Given the following set of data, use regression to find a polynomial that best fits the data. Determine what percentage of the variation can be explained for. Try 1^{st} order, 2^{nd} order … until you get a good fit. Hint: put the data into two matrices so that you can easily manipulate them using MATLAB.

$$\sum_{i=1}^{n} x_i \qquad \sum_{i=1}^{n} (x_i)^2$$

x =	f(x) =
-10	-436.881
-8	-235.656
-6	32.202
-4	57.553
-2	199.601
0	200
2	168.399
4	214.447
6	79.798
8	123.656
10	36.881
12	14.359
14	73.495
16	25.506
18	205.1
20	274.682

2. Given the following set of data, use regression to find a multi dimensional polynomial shape that best fits the data. Use a 2nd order 2 dimensional fit. The data points are stored in a file called:

"hw5points.txt"

available from the HW web site. The 1st column are the x values, the 2nd column are the y values and the 3rd are the z or f(x,y) values. Or you can regenerate the data points using the following function:

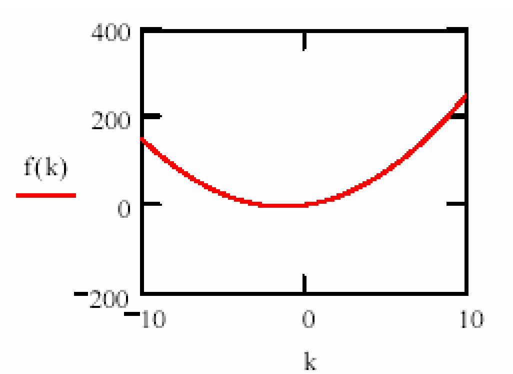
$$f(x,y) = 2x^2 + y^2 + 3y + 4$$

If you regenerate the data select at least 100 points from the range of -10 to 10 for both x and y.

Note: You will find the best fit to be the function above with a 100% fit. <u>DO NOT use this function anywhere in your work.</u> Use it only to generate the data points. You must use regression to find the function.

3. For the set of data points, find an interpolating polynomial that fits the data. You may use either Newton's or Lagrange interpolating polynomial. Do this by hand. Estimate the value at x = 5 and find the error. $f(x) = 2x^2 + 5x + 3$

x =	f(x) =
-10	153
0	3
10	253



4. Integrate the function above, $f(x) = 2x^2 + 5x + 3$, from -5 to 10 using multiple applications of the 1/3 rule and the 3/8 rule if needed. Estimate the error using the 1/3 rule estimator and compare to the actual error using $f(x) = 2x^2 + 5x + 3$.

   Use the pints:

x =	f(x) =
-5	28
-4	15
-3	6
-2	1
-1	0
0	3
1	10
2	21
3	36
4	55
5	78
6	105
7	136
8	171
9	210
10	253