

Practice problems for Test 2

1. In each case below, tell whether the variable a in the final step is restricted or unrestricted.

- | | |
|---|---|
| a) 1. $\forall x \exists y P(x, y)$ pr. | b) 1. $\exists x \forall y P(x, y)$ pr. |
| 2. $\exists y P(a, y)$ UI, 1 | 2. $\forall y P(c, y)$ EI, 1 |
| 3. $P(a, b)$ EI, 2 | 3. $P(c, a)$ UI, 2 |
| c) 1. $\exists x \forall y P(x, y)$ pr. | d) 1. $\forall x P(x)$ pr. |
| 2. $\forall y P(a, y)$ EI, 1 | 2. $\exists y Q(y)$ pr. |
| 3. $P(a, a)$ UI, 2 | 3. $P(a)$ UI, 2 |
| | 4. $Q(b)$ EI, 1 |
| | 5. $P(a) \wedge Q(b)$ rep. 3,4 |

2. In each case below, find an example (universe and dictionary) which proves the argument form is invalid.

- a) $\exists x P(x), \forall x [Q(x) \rightarrow P(x)] \vdash \exists x Q(x)$
- b) $\forall x [P(x) \vee Q(x)], \exists x P(x) \vdash \exists x Q(x)$
- c) $\exists x [P(x) \rightarrow Q(x)], \exists x \sim Q(x) \vdash \exists x \sim P(x)$
- d) $\exists x [P(x) \wedge \sim Q(x)] \vdash \forall x [P(x) \vee Q(x)]$
- e) $\forall x P(x) \rightarrow \forall x Q(x) \vdash \forall x \{P(x) \rightarrow Q(x)\}$

3. Demonstrate: $\forall x \sim P(x) \rightarrow \exists x Q(x), \sim \exists x P(x), \forall x [Q(x) \rightarrow R(x)] \vdash \exists x R(x)$

4. Demonstrate: $\exists x \sim Q(x, x) \rightarrow \exists x \forall y P(x, y) \vdash \exists y \forall x \sim Q(x, y) \rightarrow \exists x P(x, x)$

5. Let $R(x, y) : x$ is related to y and $L(x, y) : x$ likes y . Tell whether each statement below is a translation of $\forall x \exists y \{R(x, y) \wedge \sim L(x, y)\}$. (More than one may be correct.)

- a) Everyone dislikes all of his/her relatives.
- b) Everyone has a relative he/she doesn't like.
- c) No one likes all of his/her relatives.
- d) Everyone dislikes at least one relative.
- e) Everyone is related to someone and dislikes someone
- f) No one dislikes all of his/her relatives.

Solutions:

1. a) a is restricted b) a is unrestricted c) a is: restricted d) a is unrestricted

2. Below is an example for each (these examples are not unique).

a) $U = \{1, 2\}$ $P(x) : x = 1$ $Q(x) : x < 1$	<table border="1"> <tr><th></th><th>$P(x)$</th><th>$Q(x)$</th></tr> <tr><td>1</td><td>T</td><td>F</td></tr> <tr><td>2</td><td>F</td><td>F</td></tr> </table>		$P(x)$	$Q(x)$	1	T	F	2	F	F
	$P(x)$	$Q(x)$								
1	T	F								
2	F	F								
b) $U = \{1, 2\}$ $P(x) : x \geq 1$ $Q(x) : x < 1$	<table border="1"> <tr><th></th><th>$P(x)$</th><th>$Q(x)$</th></tr> <tr><td>1</td><td>T</td><td>F</td></tr> <tr><td>2</td><td>T</td><td>F</td></tr> </table>		$P(x)$	$Q(x)$	1	T	F	2	T	F
	$P(x)$	$Q(x)$								
1	T	F								
2	T	F								
c) $U = \{1, 2\}$ $P(x) : x \geq 1$ $Q(x) : x = 2$	<table border="1"> <tr><th></th><th>$P(x)$</th><th>$Q(x)$</th></tr> <tr><td>1</td><td>T</td><td>F</td></tr> <tr><td>2</td><td>T</td><td>T</td></tr> </table>		$P(x)$	$Q(x)$	1	T	F	2	T	T
	$P(x)$	$Q(x)$								
1	T	F								
2	T	T								
d) $U = \{1, 2\}$ $P(x) : x = 2$ $Q(x) : x < 1$	<table border="1"> <tr><th></th><th>$P(x)$</th><th>$Q(x)$</th></tr> <tr><td>1</td><td>F</td><td>F</td></tr> <tr><td>2</td><td>T</td><td>F</td></tr> </table>		$P(x)$	$Q(x)$	1	F	F	2	T	F
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e) $U = \{1, 2\}$ $P(x) : x = 1$ $Q(x) : x = 2$	<table border="1"> <tr><th></th><th>$P(x)$</th><th>$Q(x)$</th></tr> <tr><td>1</td><td>T</td><td>F</td></tr> <tr><td>2</td><td>F</td><td>T</td></tr> </table>		$P(x)$	$Q(x)$	1	T	F	2	F	T
	$P(x)$	$Q(x)$								
1	T	F								
2	F	T								

3. 1. $\forall x \sim P(x) \rightarrow \exists x Q(x)$ pr.
2. $\sim \exists x P(x)$ pr.
3. $\forall x [Q(x) \rightarrow R(x)]$ pr.
4. $\forall x \sim P(x)$ EQ 2, NE
5. $\exists x Q(x)$ det. 1,4
6. $Q(a)$ EI, 5
7. $Q(a) \rightarrow R(a)$ UI, 3
8. $R(a)$ det. 6,7
9. $\exists x R(x)$ EG, 8

4. 1. $\exists x \sim Q(x,x) \rightarrow \exists x \forall y P(x,y)$ pr.
2. $\exists y \forall x \sim Q(x,y)$ as.
3. $\forall x \sim Q(x,a)$ EI, 2
4. $\sim Q(a,a)$ UI, 3
5. $\exists x \sim Q(x,x)$ EG, 4
6. $\exists x \forall y P(x,y)$ det. 1,5
7. $\forall y P(b,y)$ EI, 6
8. $P(b,b)$ UI, 7
9. $\exists x P(x,x)$ EG, 8
10. $\exists y \forall x \sim Q(x,y) \rightarrow \exists x P(x,x)$ RCP

5. b), c) and d) are all translations of the given statement.